Advanced Process Control and Global Optimization for Complex Processes

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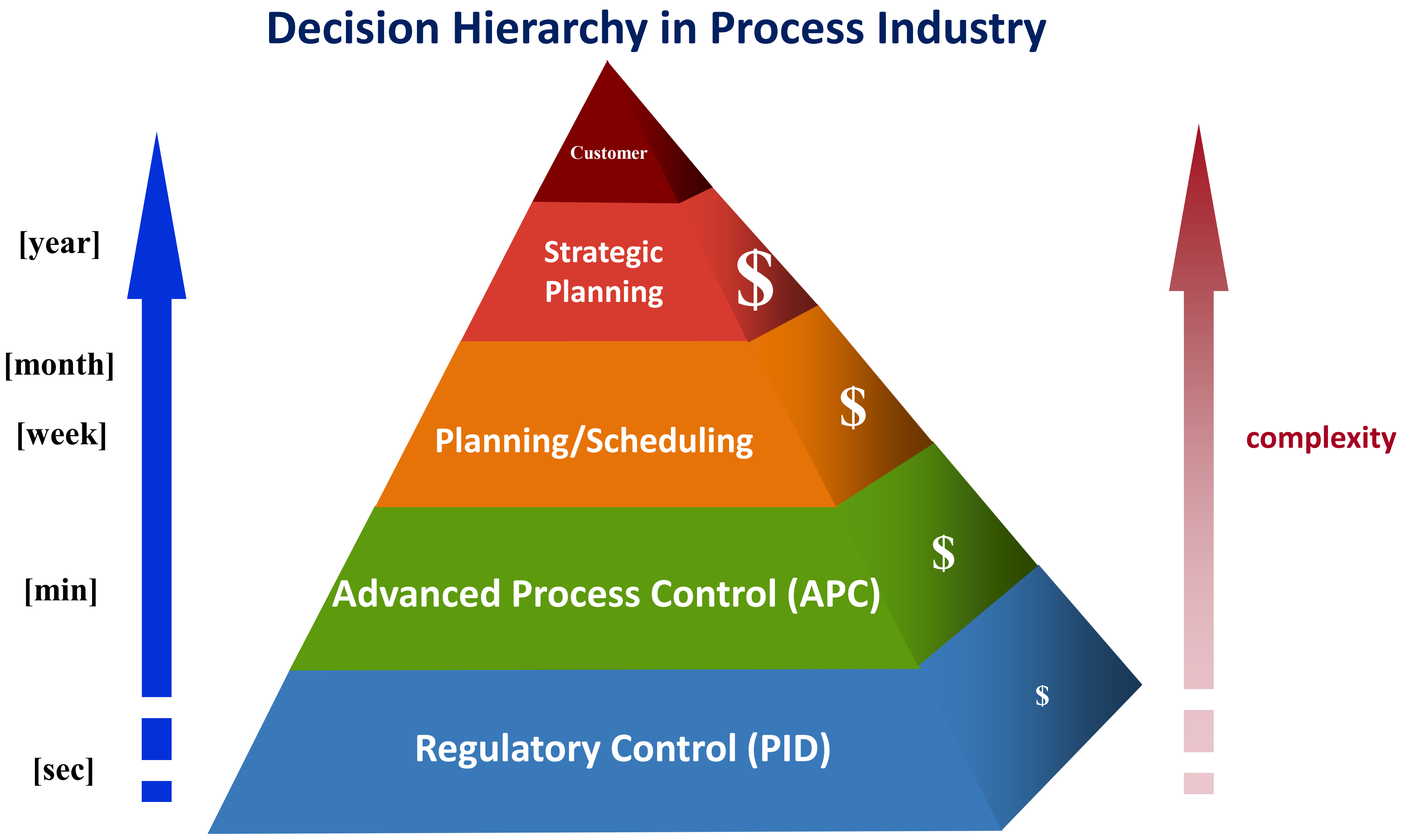
Research Scope



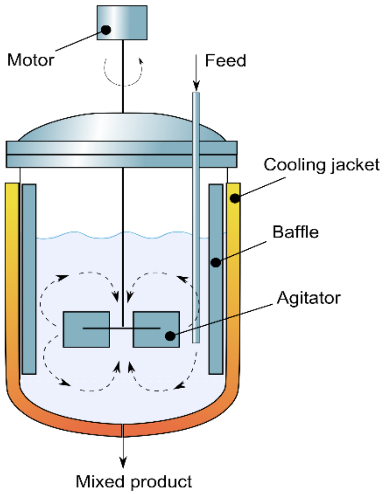
Outline

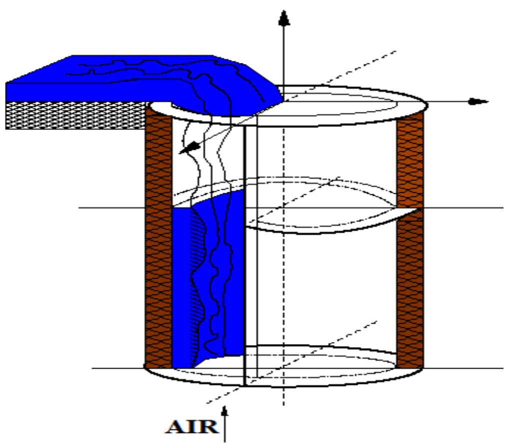
Control and Optimization of Complex Chemical Processes

Model Predictive Control for Nonlinear ODE System

Advanced Process Control for PDE System

Global Optimization under Uncertainty

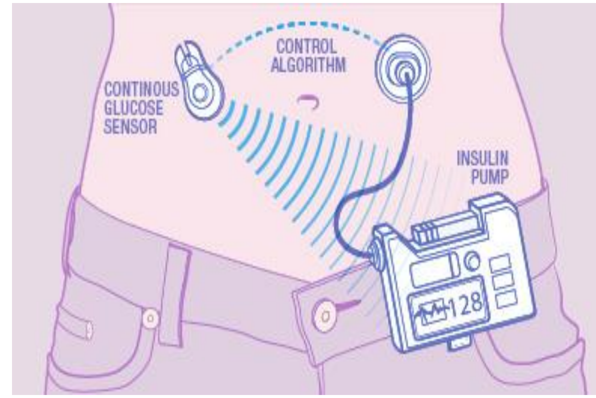




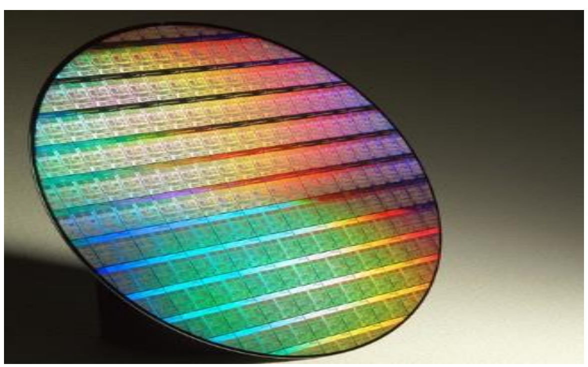
PART I : Model Predictive Control for Nonlinear ODE System

Model Predictive Control

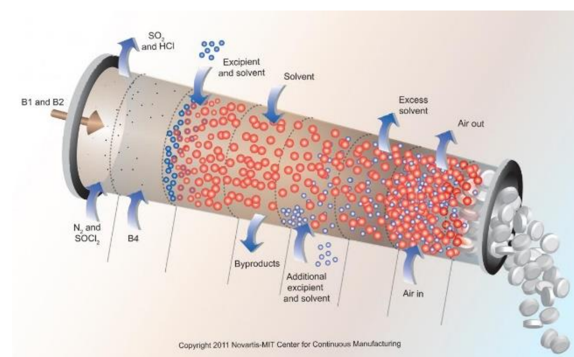
Refinery



Artificial Pancreas

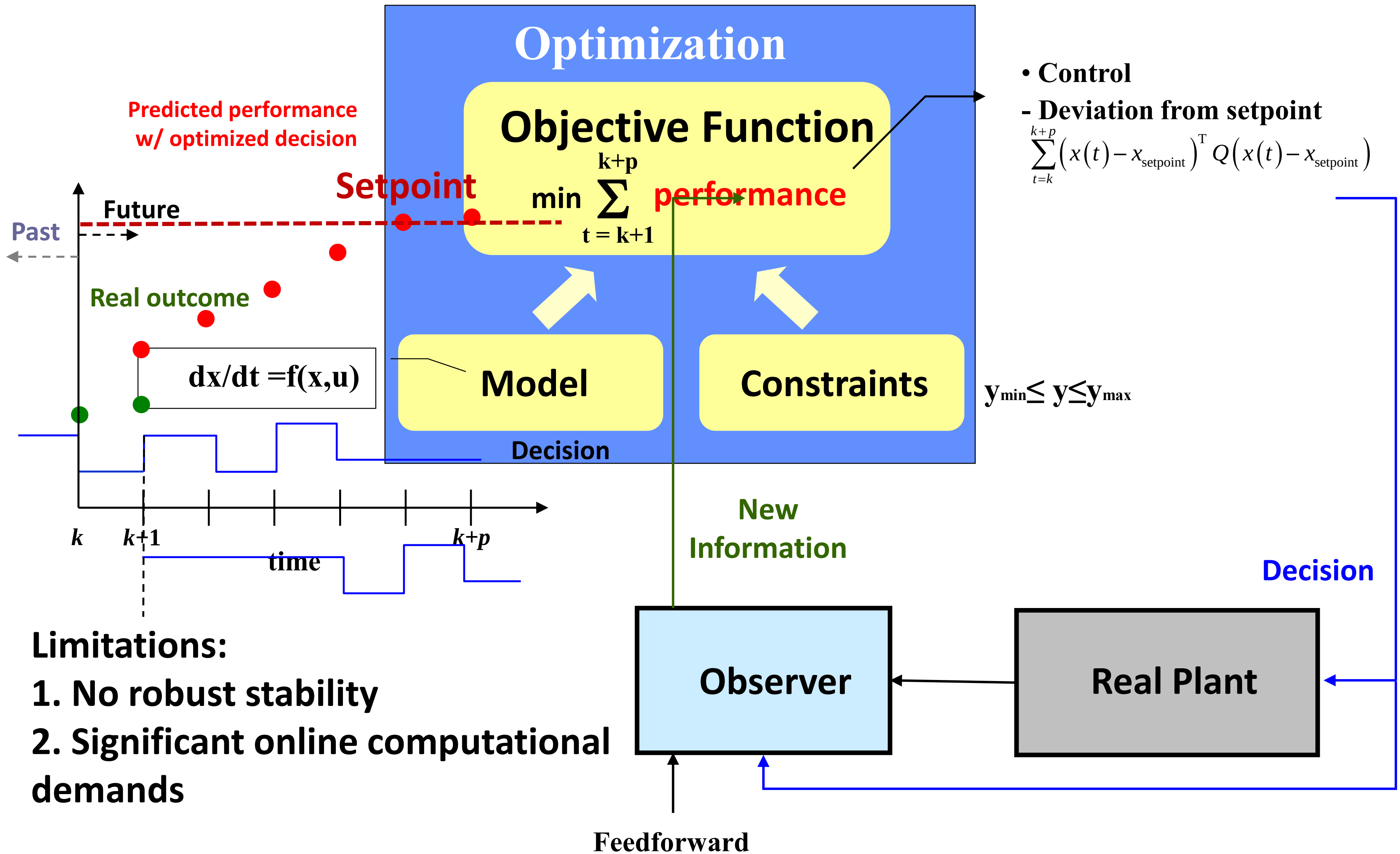


Semiconductor Manufacturing



Pharmaceutical Manufacturing

Nonlinear Model Predictive Control

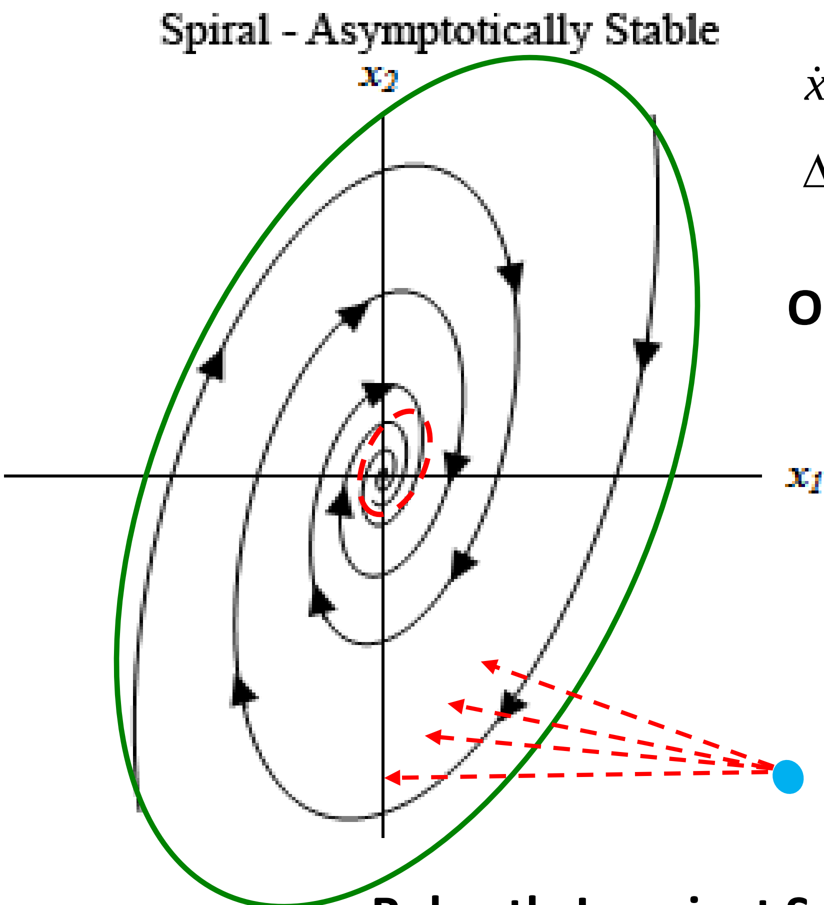


Limitations:

1. No robust stability
2. Significant online computational demands

Robust Stability

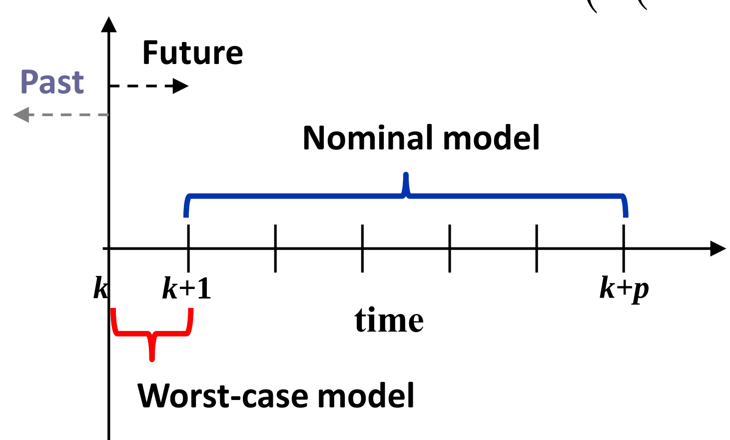
- Robust control Lyapunov function (RCLF: $V(x) = x^{\mathrm{T}} P x$)



$$\dot{x}(t) = \left(f(x) + \Delta f(x,t)\right) + \left(g(x) + \Delta g(x,t)\right)u(t)$$

$\Delta f(x,t), \Delta g(x,t)$: bounded uncertainties

One-step worst-case MPC: $V\left(x(k+1)\right) \le V\left(x(k)\right)$



RCLF forms the terminal constraint

Robustly Invariant Set: $\inf_{u \in U} \sup_{\Delta f, \Delta g} \bar{x}^T P \bullet \left(f(\bar{x}) + \Delta f + \left(g(\bar{x}) + \Delta g\right)u \right) < 0$

Robust Control Lyapunov Function

Objective: Enlarging ROA and reducing residual set

$$\max_{P}\left(\frac{\min_{\bar{x}} \bar{x}^T P \bar{x}}{\max_{\hat{x}} \hat{x}^T P \hat{x}}\right)$$

Characterize the level of ROA (numerator)

Characterize the level of residual set (denominator)

$$\text{s.t.}\ \inf_{u} \sup_{\Delta f, \Delta g} \bar{x}^T P \bullet \left(f(\bar{x}) + \Delta f + \left(g(\bar{x}) + \Delta g \right) u \right) \geq 0$$

$$u_{\min} \leq u \leq u_{\max}$$

$$P \succ 0$$

$$\hat{x} \in \Omega$$

$$\bar{x}^T P \bar{x} \geq \max_{\hat{x} \in \Omega} \hat{x}^T P \hat{x}$$

Bilevel Optimization:

Fractional programming: Dinkelbach's method

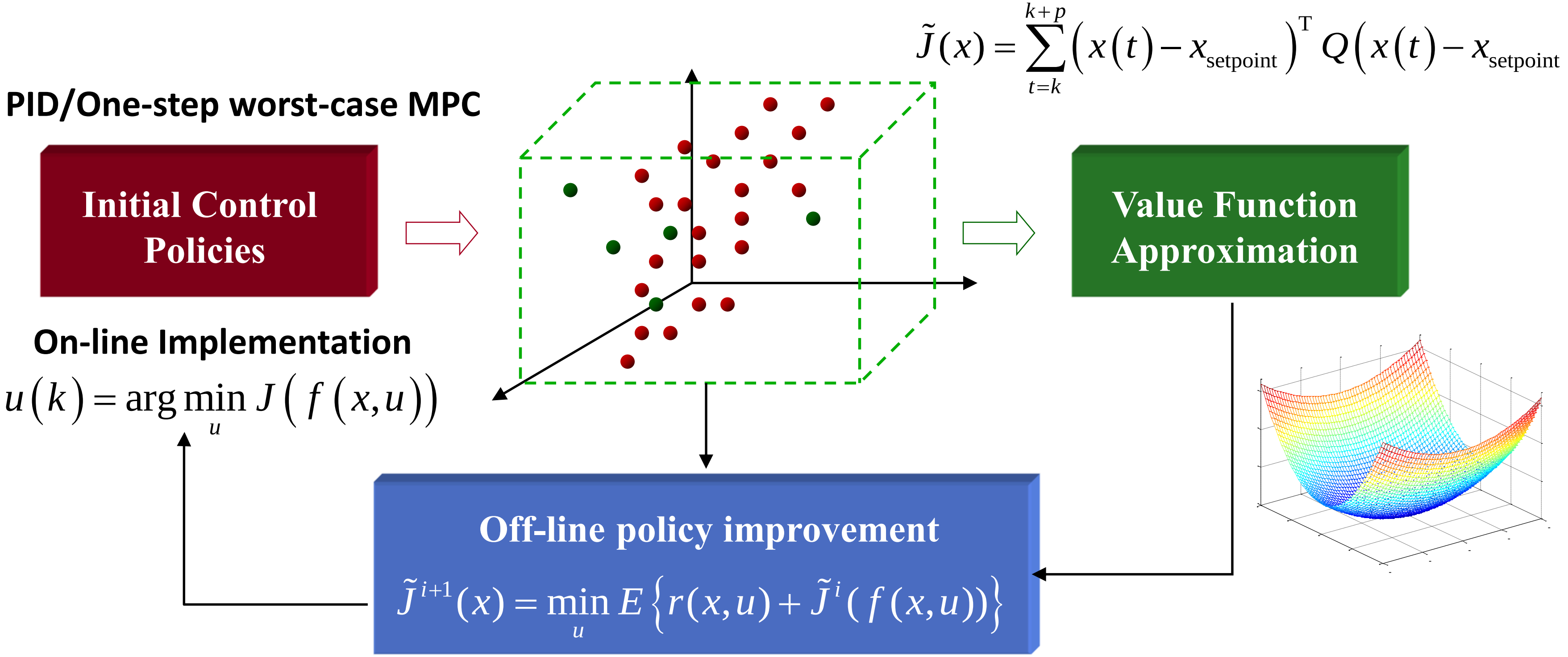
Coordinate search: Single variable optimization

Yang & Lee, IET control theory and application, 2012

Yang & Lee, Journal of Process Control, 2011

Performance Improvement

- **Approximate Dynamic Programming**



MPC: $u(k),u(k+1),\cdots,u(k+p)$

H. Nosair, Y. Yang & J. M. Lee, Control Engineering Practice, 2010
Y. Yang & J. M. Lee, Computers and Chemical Engineering, 2010

Example

Uncertainty

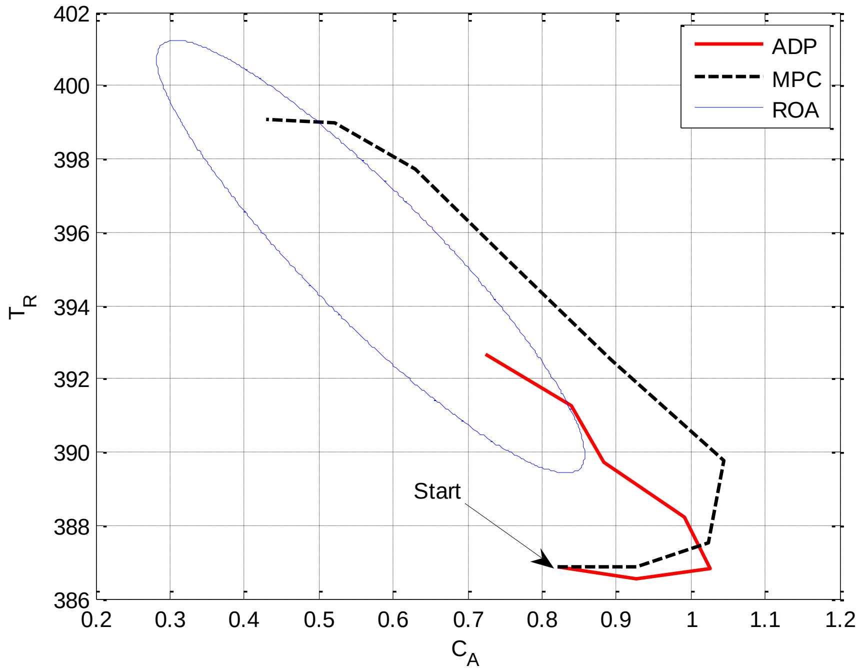
$$\dot{C}_A = \frac{F}{V}\left(C_{A0}\left(1+\varepsilon\right) - C_A\right) - k_0 e^{-E/RT_R} C_A$$

Uncertainty

$$\dot{T}_R = \frac{F}{V}\left(T_{R0} - T_R\right) - \frac{\Delta H}{\rho C_p} k_0 e^{-E/RT_R} C_A + \frac{Q_\sigma}{\rho C_p V} + \mu$$

	MPC	ADP
Average computational time	8.6s	0.1s
Average control cost	252.7	230.2

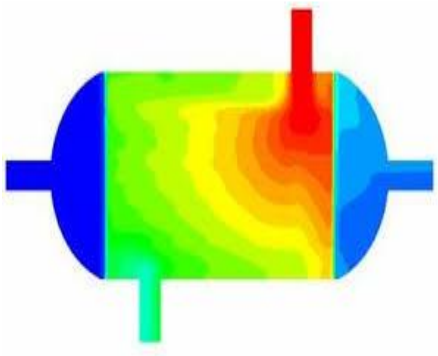
$$\sum_{t=k}^{k+p} \left(x(t) - x_{\text{setpoint}}\right)^{\mathrm{T}} Q \left(x(t) - x_{\text{setpoint}}\right)$$



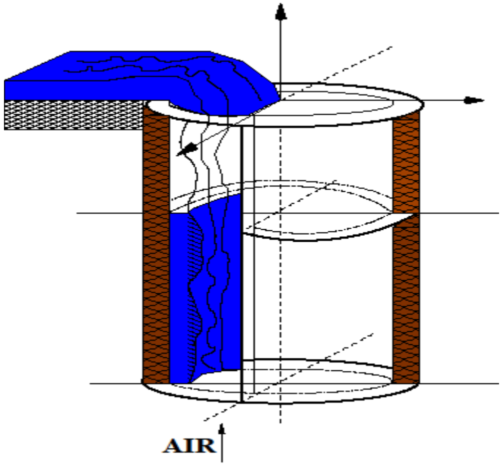
Y. Yang & J. M. Lee, Journal of Process Control, 2012
Y. Yang & J. M. Lee, Journal of Process Control, 2013

PART II : Advanced Process Control
for
PDE System

PDE Control: Motivation

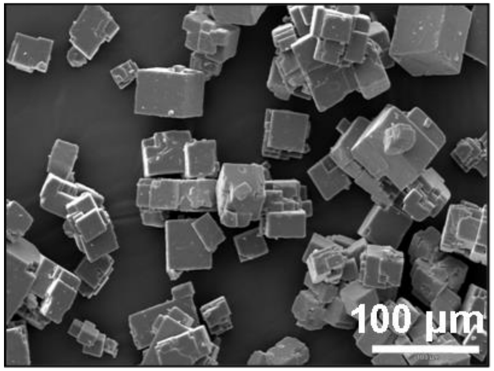


Heat equation

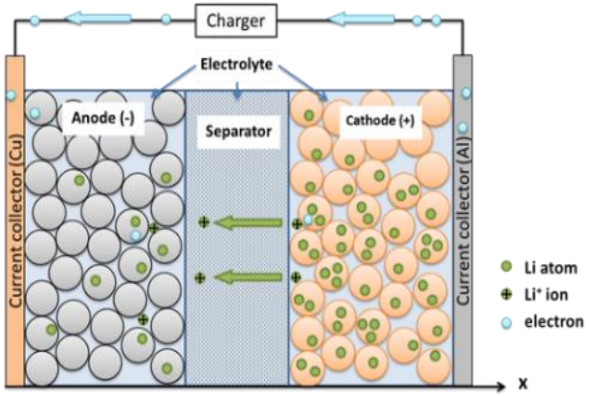


Fluid mechanics

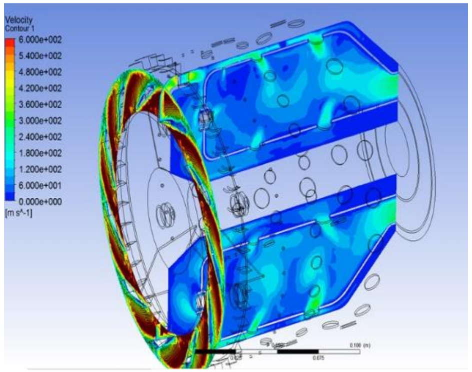
Rod pump



Crystallization



Lithium battery



Engine

PDE Control: Application

$$v^2 \frac{\partial^2 x(z,t)}{\partial z^2} = \frac{\partial^2 x(z,\mathrm{t})}{\partial t^2} + c\frac{\partial x(z,t)}{\partial z}$$

Stress difference **Hooke's law** | **Acceleration** | **Friction**

x : displacement

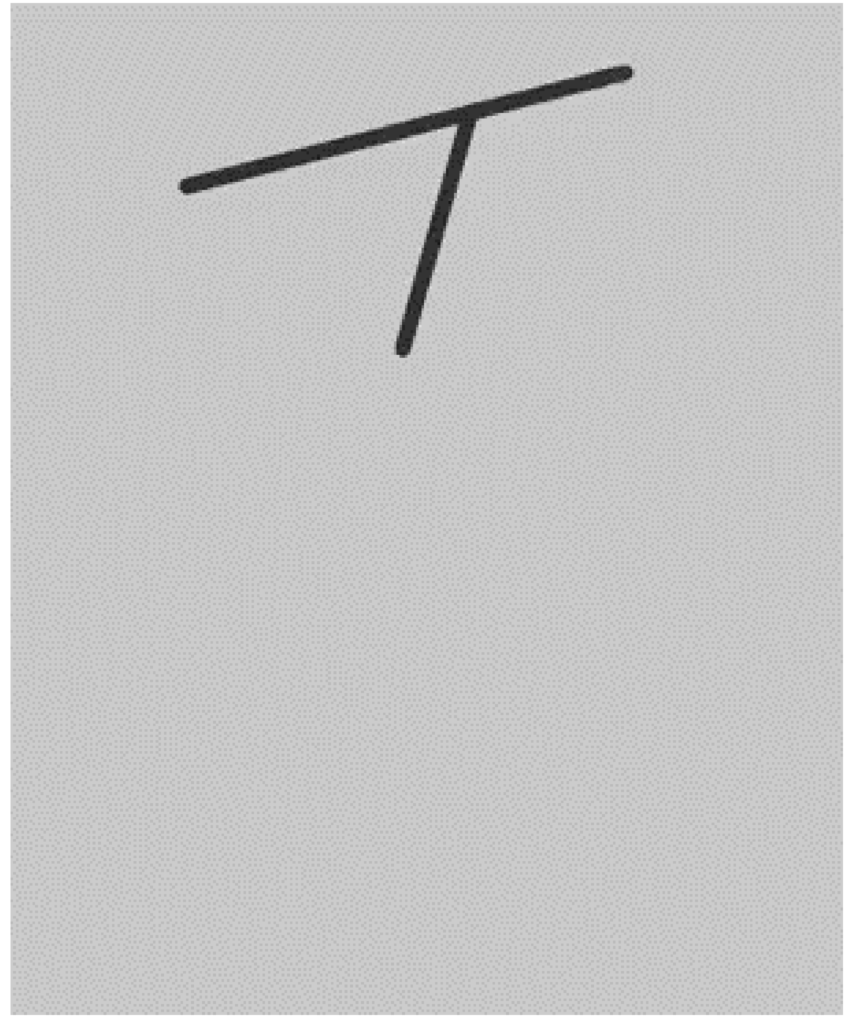
z : position

t : time

Sensor: Surface

Estimation:

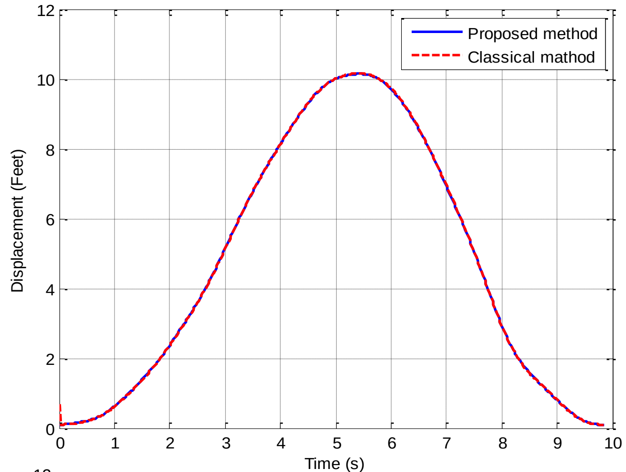
Downhole load & Displacement

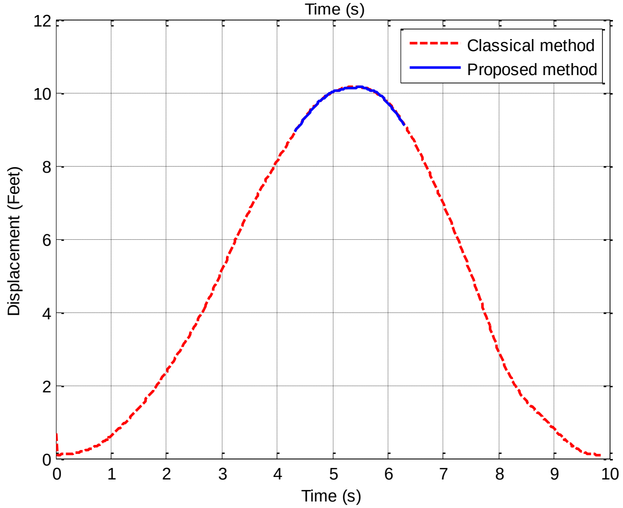


PDE Control: Application

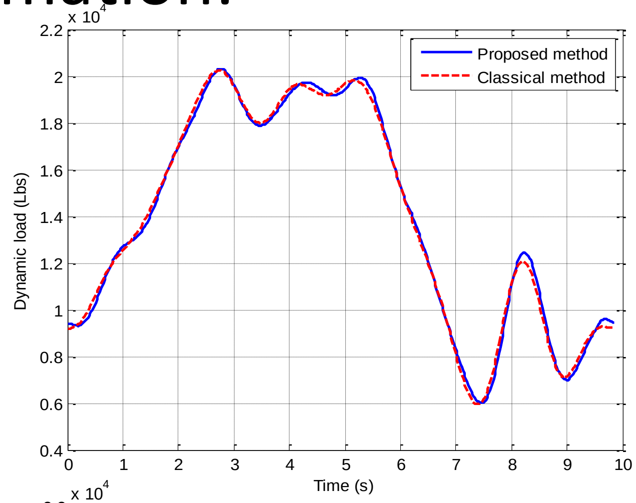
- Displacement and load estimation:

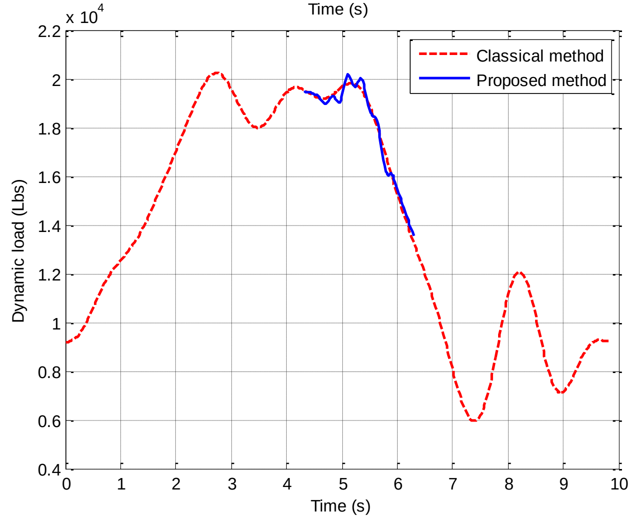
Downhole Displacement





Downhole Load





(Y. Yang & S. Dubljevic, SPE Annual Conference, 2012)

PDE Control: Model Reduction

- Spectral method:

Slow modal state

$$x(z,t) = \sum_{i=1}^{\infty} a_i(t)\phi_i(z) \xrightarrow{\text{Truncation}} x(z,t) \approx \sum_{i=1}^{n_s} a_i(t)\phi_i(z)$$

State #	FD 2nd order (max error)	FD 4th order (max error)	Fourier spectral (max error)
16	6.13e-1	2.36e-1	2.55e-4
32	1.99e-1	2.67e-2	1.05e-11
64	5.42e-2	1.85e-3	6.22e-13

- My theoretical contributions:
  - Handle the truncation residual in the model predictive control

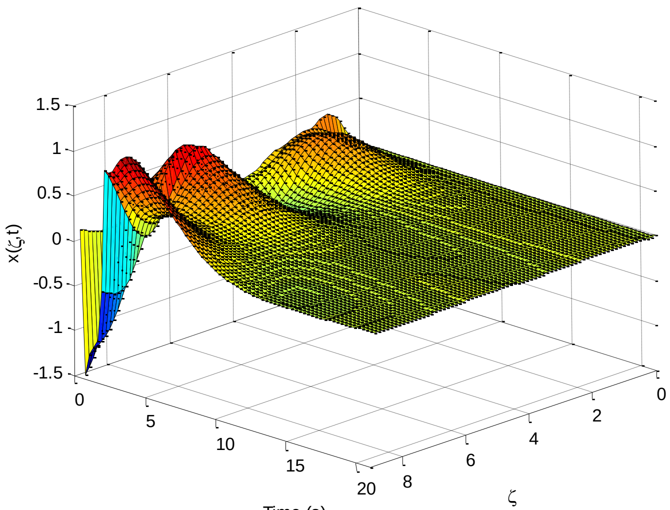
$$x(z,t) = \sum_{i=1}^{n_s} a_i(t)\phi_i(z) + R(z,t)$$

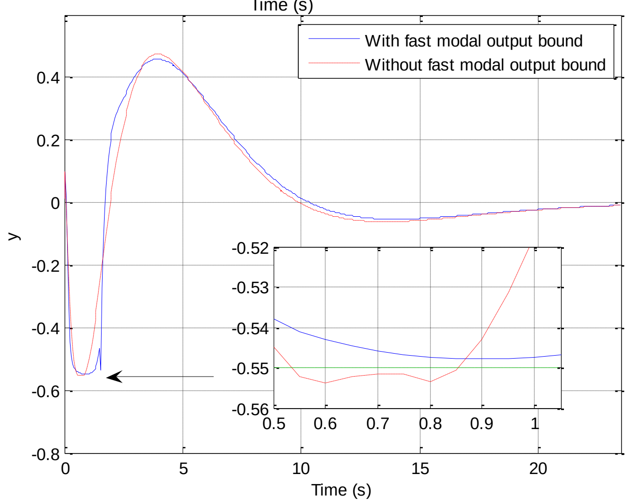
  - Controller and observer synthesis by taking residual into account

Spectral Methods in Fluid Dynamics, Canuto, 1988 (Table 1.1 1st order wave equation).

PDE Control: Model Predictive Control

- Two-phase flow (linearized K-S equation)





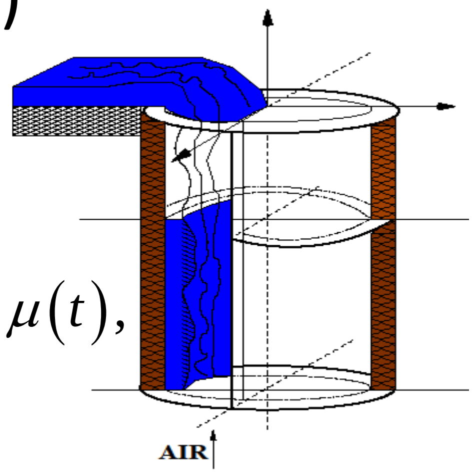
$$\frac{\partial x}{\partial t} + v\frac{\partial^4 x}{\partial \zeta^4} + \frac{\partial^2 x}{\partial \zeta^2} = 0,$$

$$x(0,t) = 0, x(l,t) = d_1 u(t) + d_2 \mu(t),$$

$$x(0,t) = 0, x(l,t) = u(t),$$

$u(t), \mu(t)$: boundary inputs,

$x(\zeta,t)$: variance of the thin film thickness.



Spectral method
Galerkin's model reduction
MPC (Full state feedback)

(Y. Yang & S. Dubljevic, Journal of process control, 2013)

PDE Control: Controller & Observer

- Controller and observer synthesis:

$$\hat{a}_u(k+1) = \Lambda_u \hat{a}_u(k) + \mathfrak{B}_u \tilde{u}(k) + L_u\left(y(k) - y_s(k) - \hat{y}_u(k)\right)$$

$$a_s(k+1) = \Lambda_s a_s(k) + \mathfrak{B}_s \tilde{u}(k)$$

$$\tilde{u}(k+1) = K_u \hat{a}_u(k) + G_u u(k)$$

$$a = \left[a_1, a_2, \cdots, a_{n_u}, \,\middle|\, \underbrace{a_{n_u+1}, a_{n_u+2}, \cdots, a_{n_u+n_s},}_{\text{Slow modal states}} \,\vdots\, a_{n_u+n_s+1}, a_{n_u+n_s+2}, \cdots\right]$$

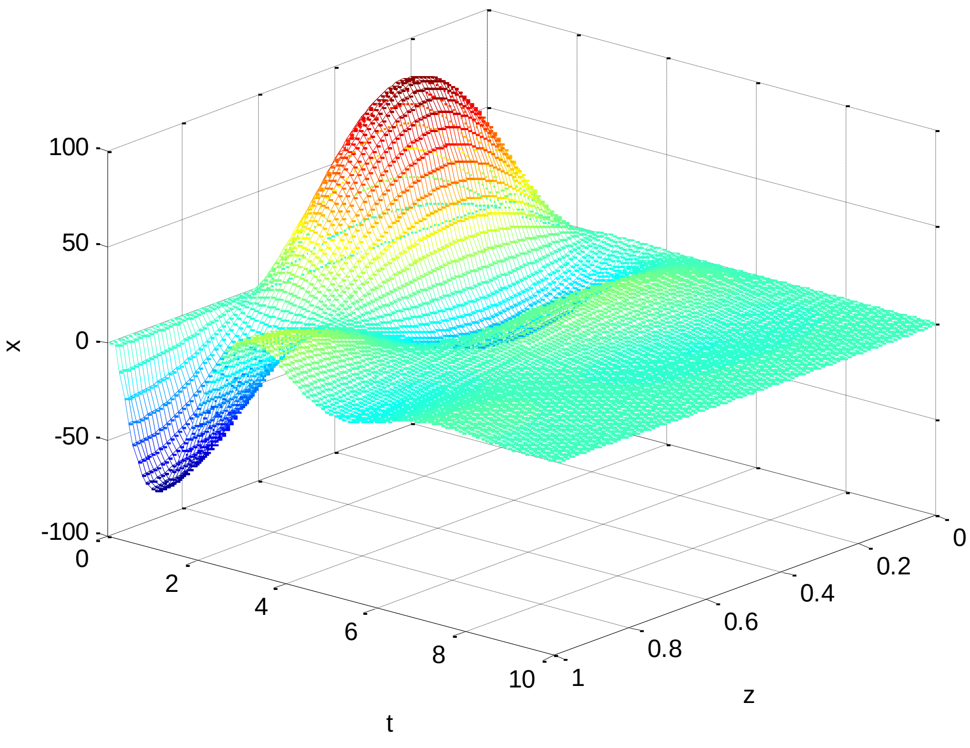
Slow modal states

Questions

1. **How to find the order of slow modal states?**
2. **How to design the feedback law and observer gains to obtain a closed-loop system with large enough region of attraction?**

PDE Control: Controller & Observer

◆ Results (2nd order Parabolic PDE Stabilization)



$$\frac{\partial x}{\partial t}(z,t) = b\frac{\partial^2 x}{\partial z^2}(z,t) + c\frac{\partial x}{\partial z}(z,t) + dx(z,t)$$

$$x(0,t) = u(t)$$

$$\frac{\partial x}{\partial z}(1,t) = 0$$

$$y(t) = \int_0^1 x(z,t)\cdot\delta(z-1)\,dz$$

Boundary sensor

$$z \in [0,1]$$

Observer
Spectral method
Galerkin's model reduction
Output feedback controller (LMI)

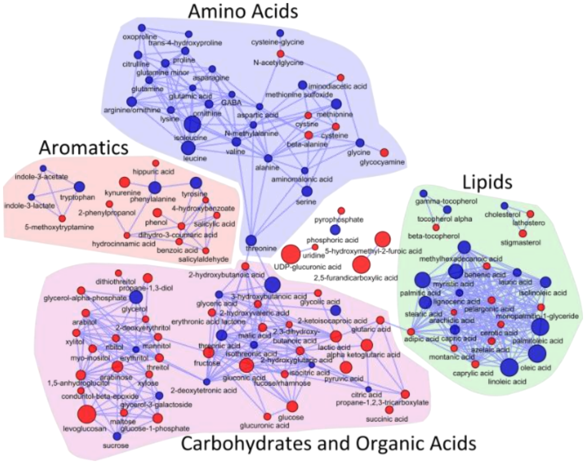
(Y. Yang & S. Dubljevic, European journal of control, 2014)

PART III : Global Optimization under Uncertainty

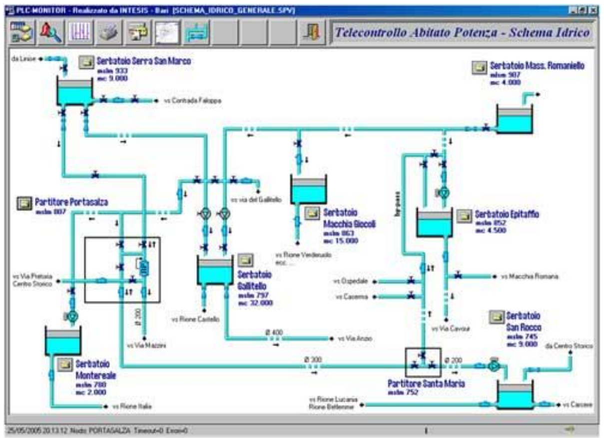
Massachusetts
Institute of
Technology

bp

Motivation

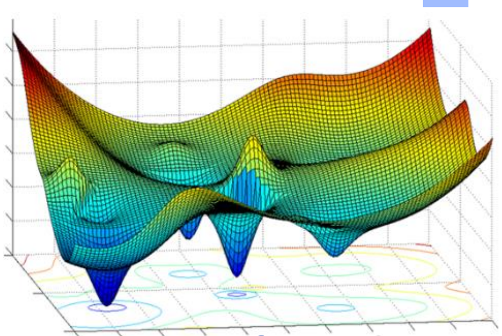


Metabolic network



Water network

Refinery

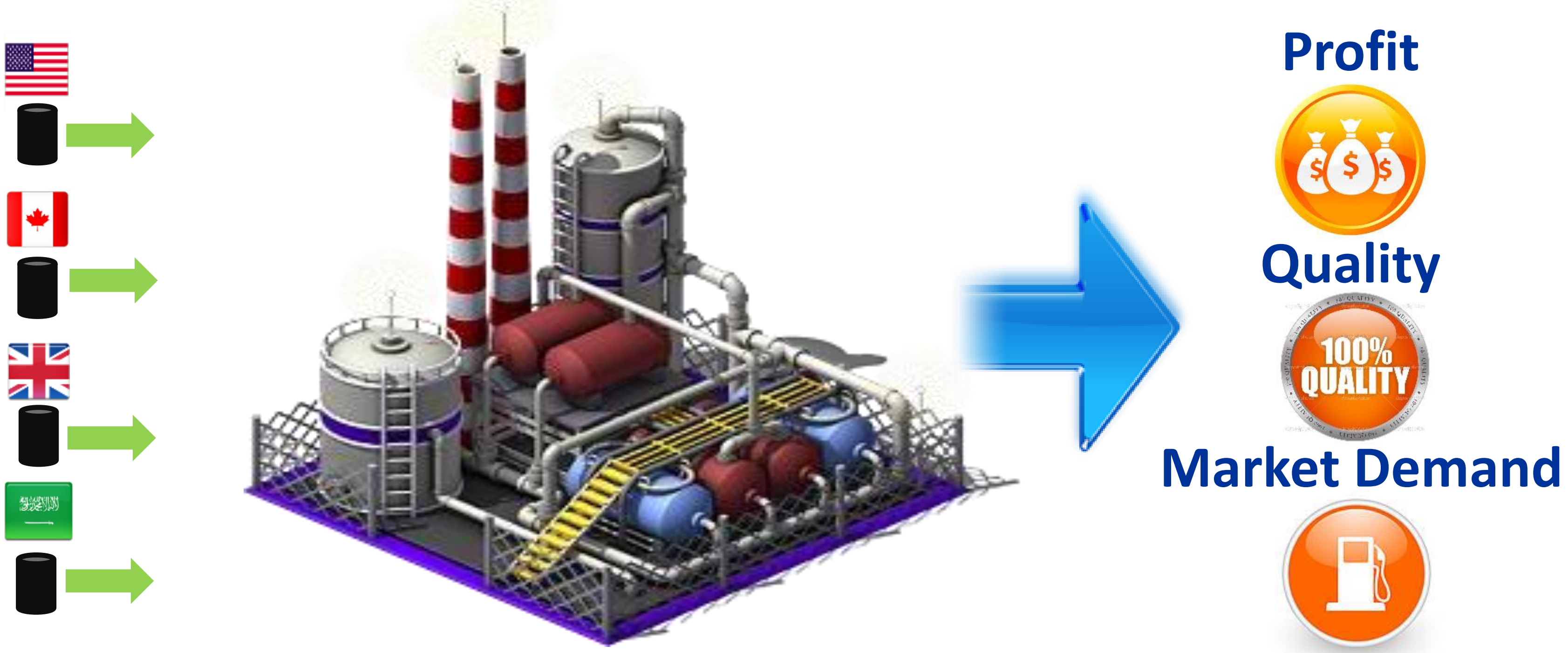


$$
\begin{aligned}
\min_{x,y}\ & c^{\mathrm{T}} x \\
\text{s.t. } & Ax \le b, \\
& F(x,y) \le d, \\
& x \in \Re,\ y \in \{0,1\}.
\end{aligned}
$$

Global Optimization: mixed-integer nonlinear programming (MINLP)

Overview

Refinery Optimization under Uncertainty

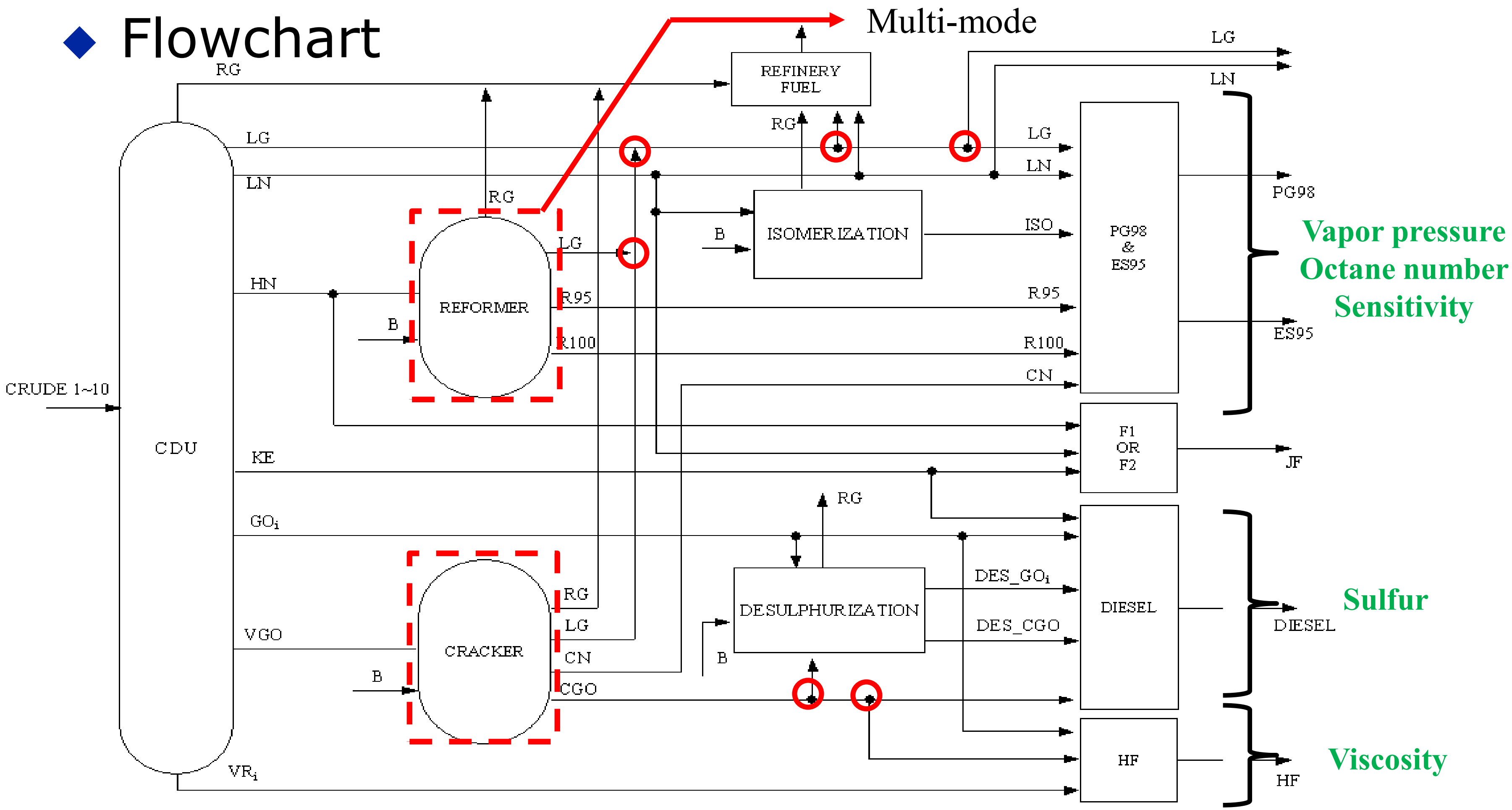


- Objective: decide the optimal crude oil procurement and refinery operations to maximize the profit, meet market demands and satisfy quality specifications.

(U.S. Patent Pub. No: US2016/0140448 A1)

Model 1

- Flowchart



○ Blending & Splitting (Pooling problem) $z = x \cdot y$

J. P. Favennec, Refinery operation and management, Editions TECHNIP, 2001

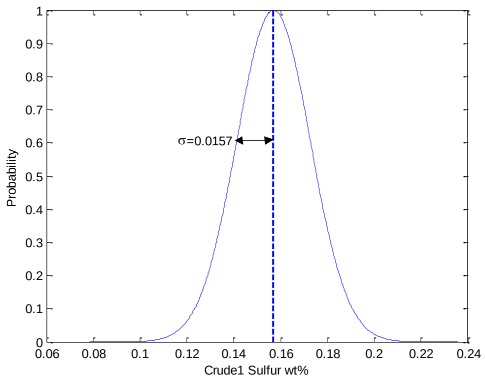
Model 1: Uncertainties

- Uncertainties:

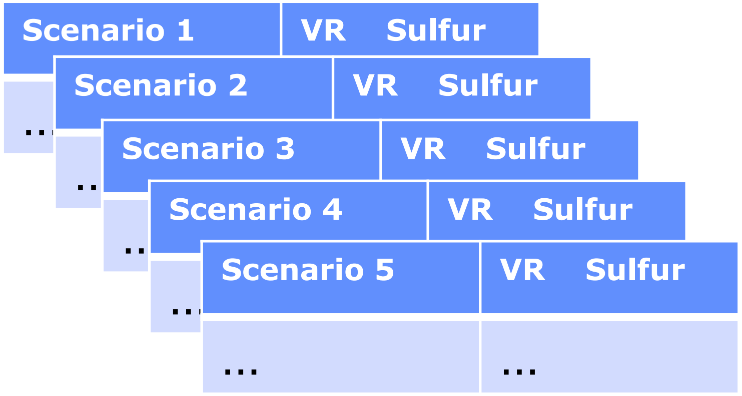
Crude	VR yield	Sulfur
Skarv	10.3%	0.209%
Saturno	26.7%	0.373%
Plutonio	21.6%	0.2%
Brent	13.6%	0.253%
Hungo	25.5%	0.321%
Polvo	37.2%	0.741%
Zakum	10.9%	0.966%
Basra	26.2%	1.826%
Thunder	20.1%	0.416%
Mars	27.6%	1.216%

Crude oil yields: www.bp.com

Gaussian Distribution:
Mean=nominal value
Standard deviation=0.1×nominal value



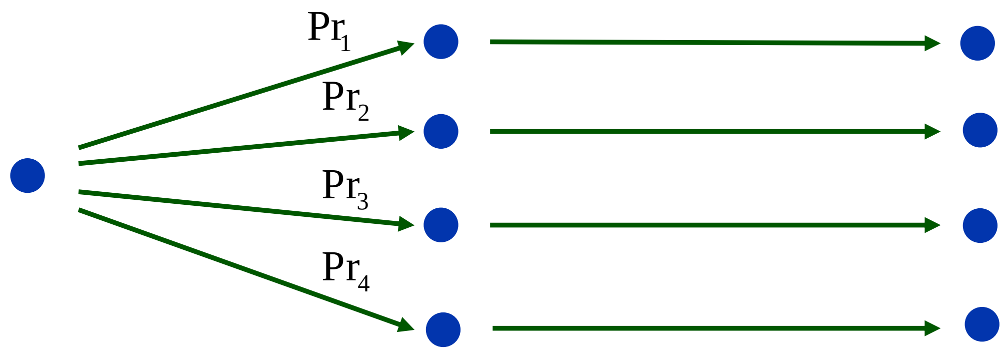
Sampling: 120 scenarios with different VR yield & sulfur content



Solution: Stochastic Programming

- Stochastic programming vs. Deterministic method:

Stage I	**Uncertainties**	**Stage II**
Crude oil Selection & purchase	**Crude oil real quality (TBP)**	**Refinery operations: Cut point temperature, Flow rate**



Pr_i : Probability of scenario i

x_d : Decison of deterministic method

x_s : Decison of stochastic method

Deterministic method:

$$x_d^* = \max_{x_d} \; -\mathrm{Cost}(x_d) + \mathrm{Profit}_{\mathrm{Nominal}}(x_d)$$

Stochastic method:

$$x_s^* = \max_{x_s} \; -\mathrm{Cost}(x_s) + \sum_i \mathrm{Profit}_i(x_s) \cdot \mathrm{Pr}_i$$

Solution: Stochastic Programming

- Two-stage Stochastic Formulation (Model 1):

$$\min \quad \text{cost of crude oil purchase} \quad - \quad \mathbb{E}(\text{product sales} - \text{operation costs})$$

s.t. Crude oil source restrictions, Material balance, Unit capacity limits, Demand requirements, Quality constraints, Pooling equations

10 Crude oil

Discretization of each crude oil quantity:
5000 barrels = 1 lot

120 Scenarios: different VR yields and sulfur concentration

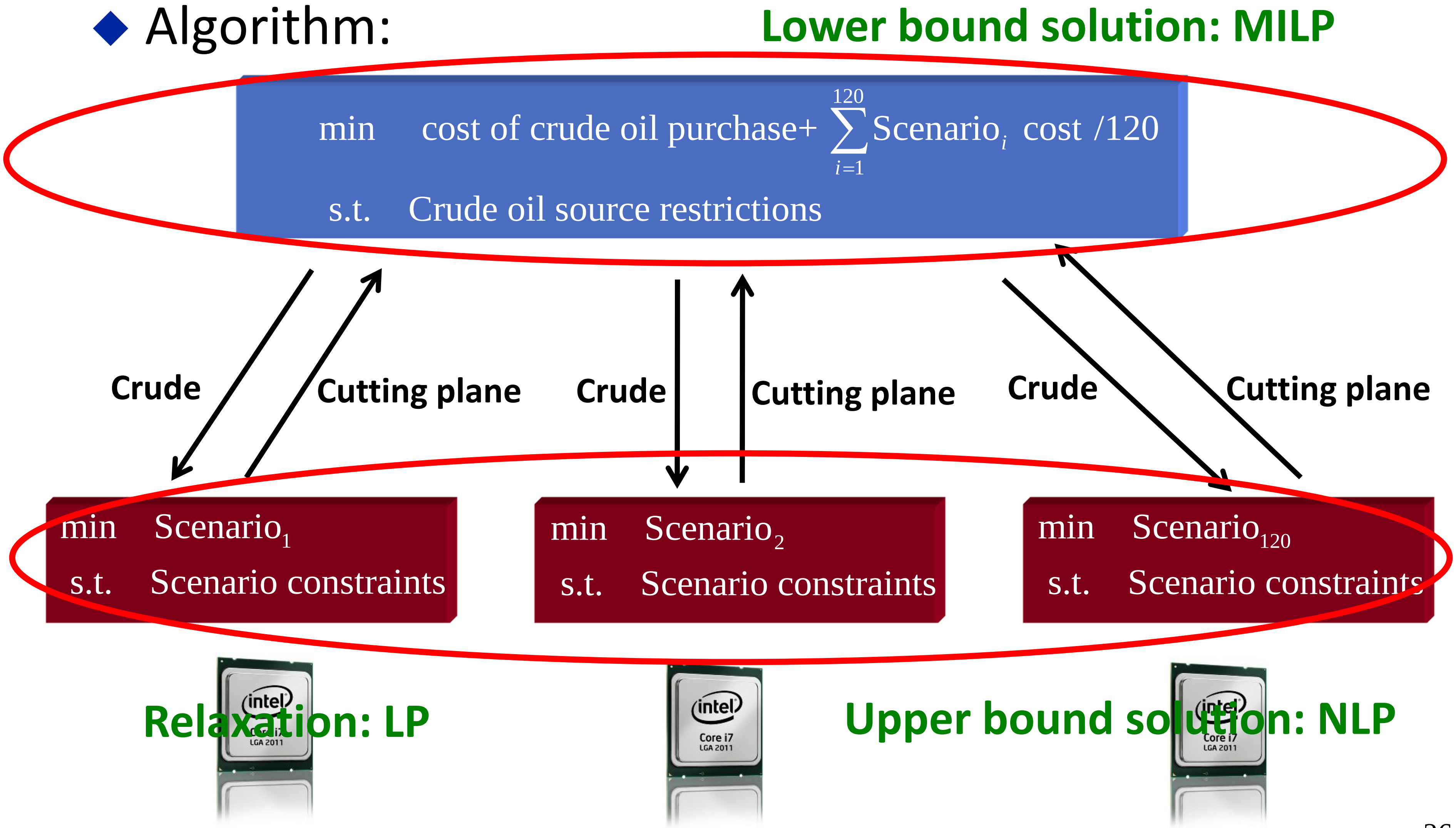
- **Model (120 scenarios):**
  13061 constraints,
  100 binary variables
  13800 continuous variables
  2760 bilinear terms

We use the non-convex generalized Benders decomposition (NGBD).

(Yang & Barton, AIChE Journal, 2016)

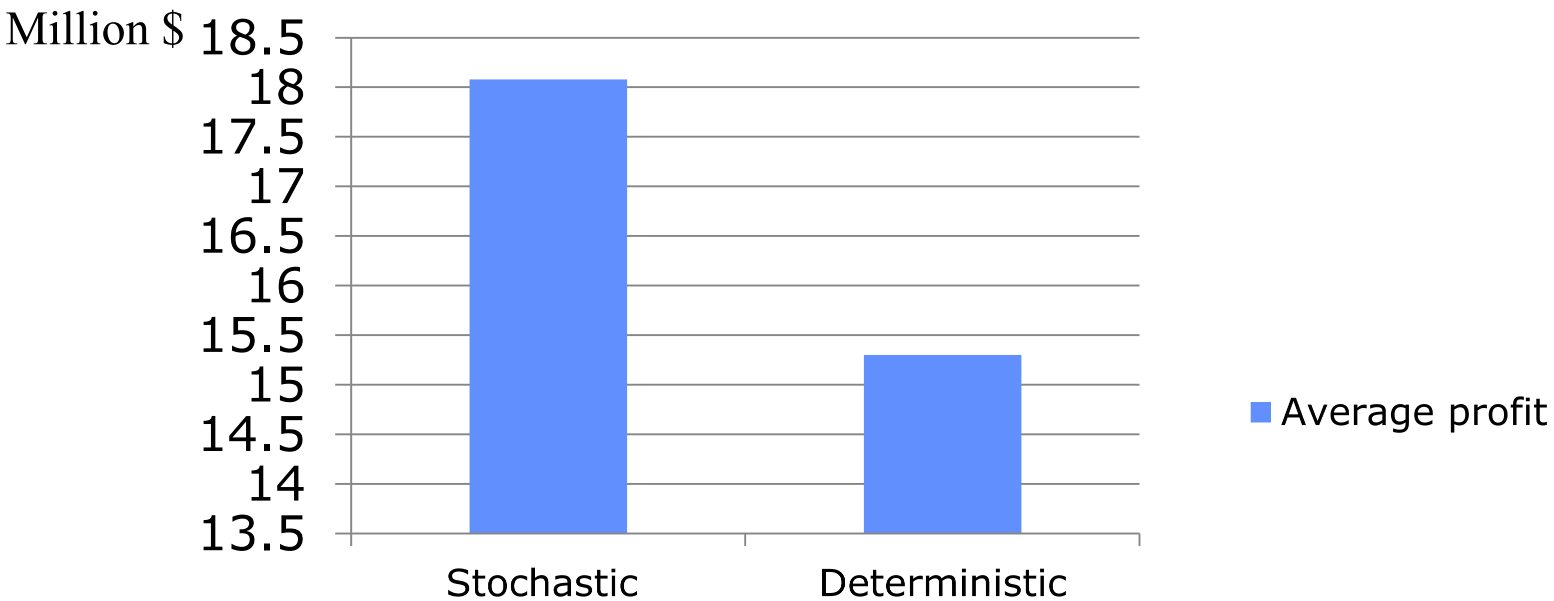
Method: NGBD

- Algorithm:



Results: Profit/Time

◆ Profit/month (18% improvement)



◆ Computational Time

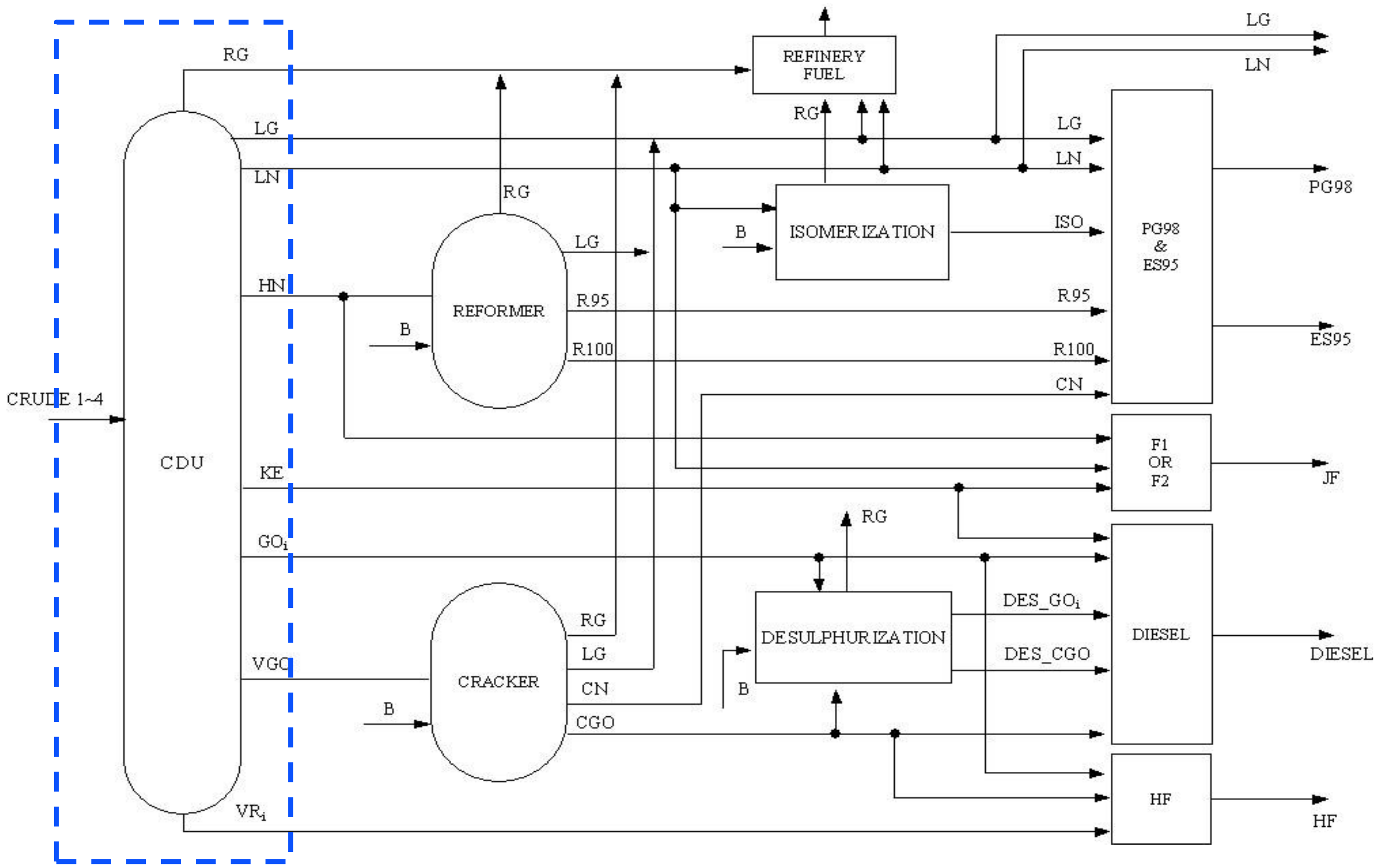
	GAMS[1]	C++[2]	(BARON, Antigone)
Total time	12.2 hours	10 mins	No solution in 1 day

1. General Algebraic Modeling System (*GAMS*)

2. Joint work with Rohit Kannan.

Model 2: Flowchart

- Refinery flowchart



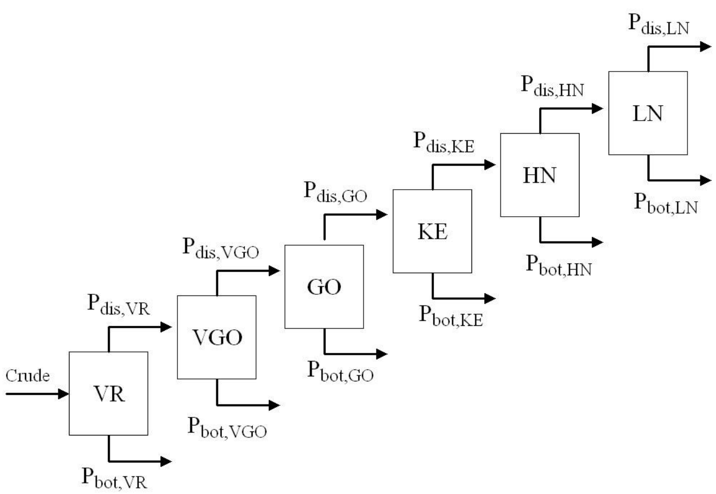
J. P. Favennec, Refinery operation and management, Editions TECHNIP, 2001

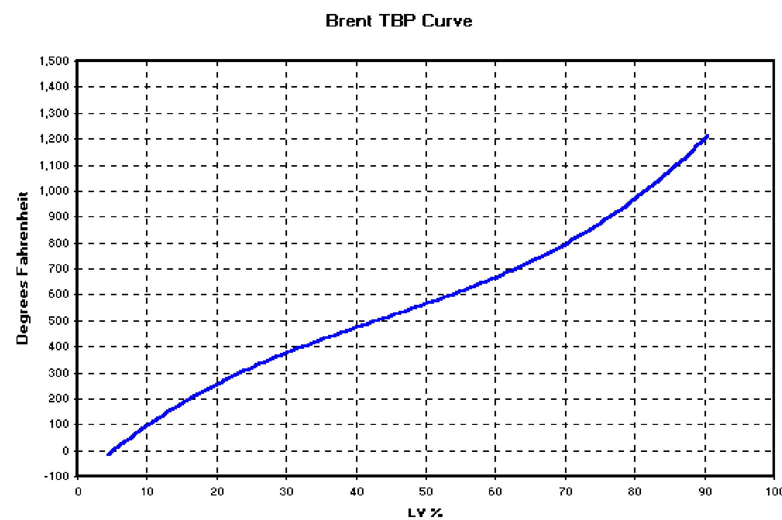
Model 2: CDU Model

- Decision variables: cut point temperature $T_j \in \left[\underline{T}_j, \bar{T}_j \right]$

$P_{\text{dis,VR}}$: Total mole flowrate in the distillate of VR cut

$P_{\text{bot,VR}}$: Total mole flowrate in the bottom (yield) of VR cut.





Crude oil TBP → **Pseudocomponent** → **Vapor pressure Equilibrium ratio** → FI equation $P_{\text{bot, j}}, P_{\text{dis, j}}$

A. M. Alattas, I. E. Grossmann and I. Palou-Rivera, I&EC research, 2011, 2012.

Model 2: CDU Model

- Fractionation index (FI) equation

$$\frac{x_{\mathrm{dis},j,i}}{x_{\mathrm{bot},j,i}} = \left(K_{j,i}\left(T_j\right)\right)^{FI}$$

$x_{\mathrm{dis},j,i}$: Mole fraction of pseudocomponent i in the distillate of cut j

$x_{\mathrm{bot},j,i}$: Mole fraction of pseudocomponent i in the bottom of cut j

$$FI = \begin{cases} FI_{r,j} & \text{if } T_{j,\mathrm{ini}} \le Tb_i < T_j \\ FI_{s,j} & \text{if } T_{j,\mathrm{end}} \ge Tb_i \ge T_j \end{cases},$$

Tb_i : true boiling point of pseudocomponent i

$$\Rightarrow \quad \frac{x_{\mathrm{dis},j,i}}{x_{\mathrm{bot},j,i}} = \left(K_{j,i}\left(T_j\right)\right)^{FI_{r,j}} v_i + \left(K_{j,i}\left(T_j\right)\right)^{FI_{s,j}} \left(1 - v_i\right), \quad v_i \in \{0,1\},$$

$$K_{j,i}\left(T_j\right) = \frac{\mathrm{VP}_i\left(T_j\right)}{P},$$

VP : vapor pressure, K : equilibrium constant

Gilbert, R.J.H., AIChE Journal, 1966.

Model 2: CDU Model

- Vapor pressure surrogate model

$$\mathrm{VP}\left(Tr_{j,i}\right)$$

$$= Pc \times \exp\left(\frac{-5.96\left(1-Tr_{j,i}\right)+1.18\left(1-Tr_{j,i}\right)^{1.5}-0.56\left(1-Tr_{j,i}\right)^{3}-1.32\left(1-Tr_{j,i}\right)^{6}}{Tr_{j,i}}\right)$$

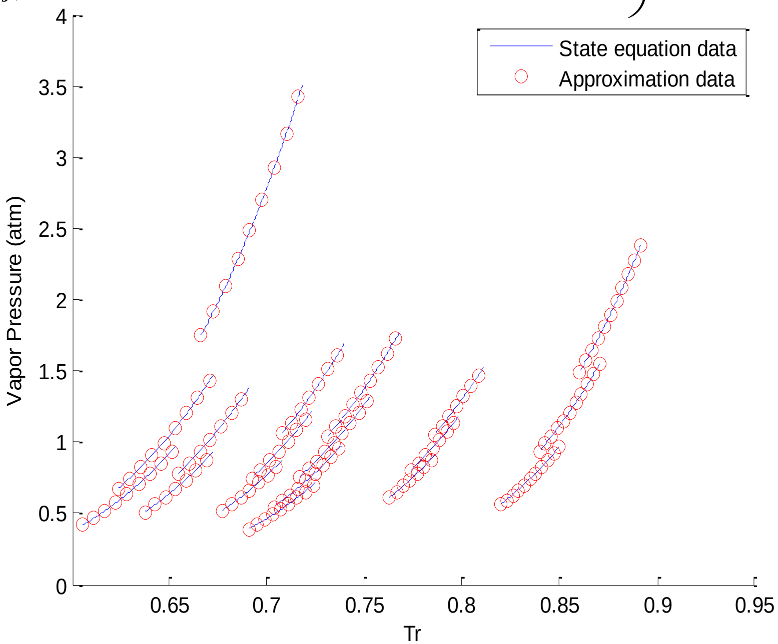
$$\times \exp\left(\omega \frac{-4.79\left(1-Tr_{j,i}\right)+0.41\left(1-Tr_{j,i}\right)^{1.5}-8.91\left(1-Tr_{j,i}\right)^{3}-4.98\left(1-Tr_{j,i}\right)^{6}}{Tr_{j,i}}\right)$$

where $Tr_{j,i} = \dfrac{T_j}{T_{ci}}$, j : cut,

i : pseudocomponent, Pc : parameter

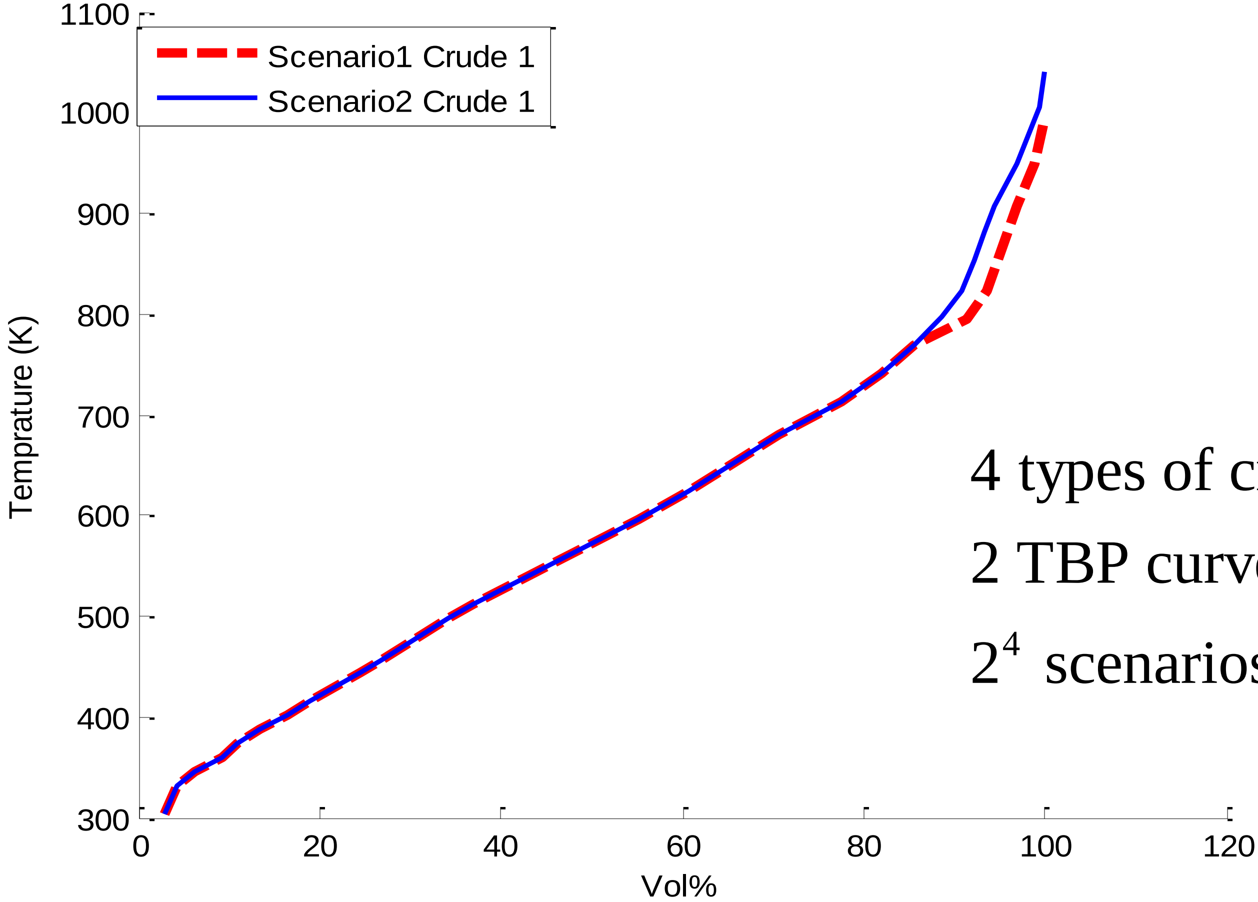
$$T_j \in \left[\underline{T}_j, \overline{T}_j\right]$$

$$VP\left(Tr_{j,i}\right) = \gamma_2^{j,i}\left(Tr_{j,i}\right)^2 + \gamma_1^{j,i} Tr_{j,i} + \gamma_0^{j,i}$$



Model 2: Uncertainty

- Uncertainty is in the heavy part of true boiling point (TBP) curve



4 types of crude oil,

2 TBP curves for each:

2^4 scenarios

Solution: Stochastic Programming

- Two-stage Stochastic Formulation (Model 2):

$$\min \ \text{cost of crude oil purchase} + \mathbb{E}\left(\text{operation costs} - \text{product sales}\right)$$

s.t. Crude oil source restrictions, Material balance,

Unit capacity limits,

CDU model,

Demand requirements,

Quality constraints,

Pooling equations

4 Crude oil

Discretization of each crude oil quantity:
5000 barrels = 1 lot

- **Model (16 scenarios):**

10058 constraints,

168 binary variables (128 in CDU model)

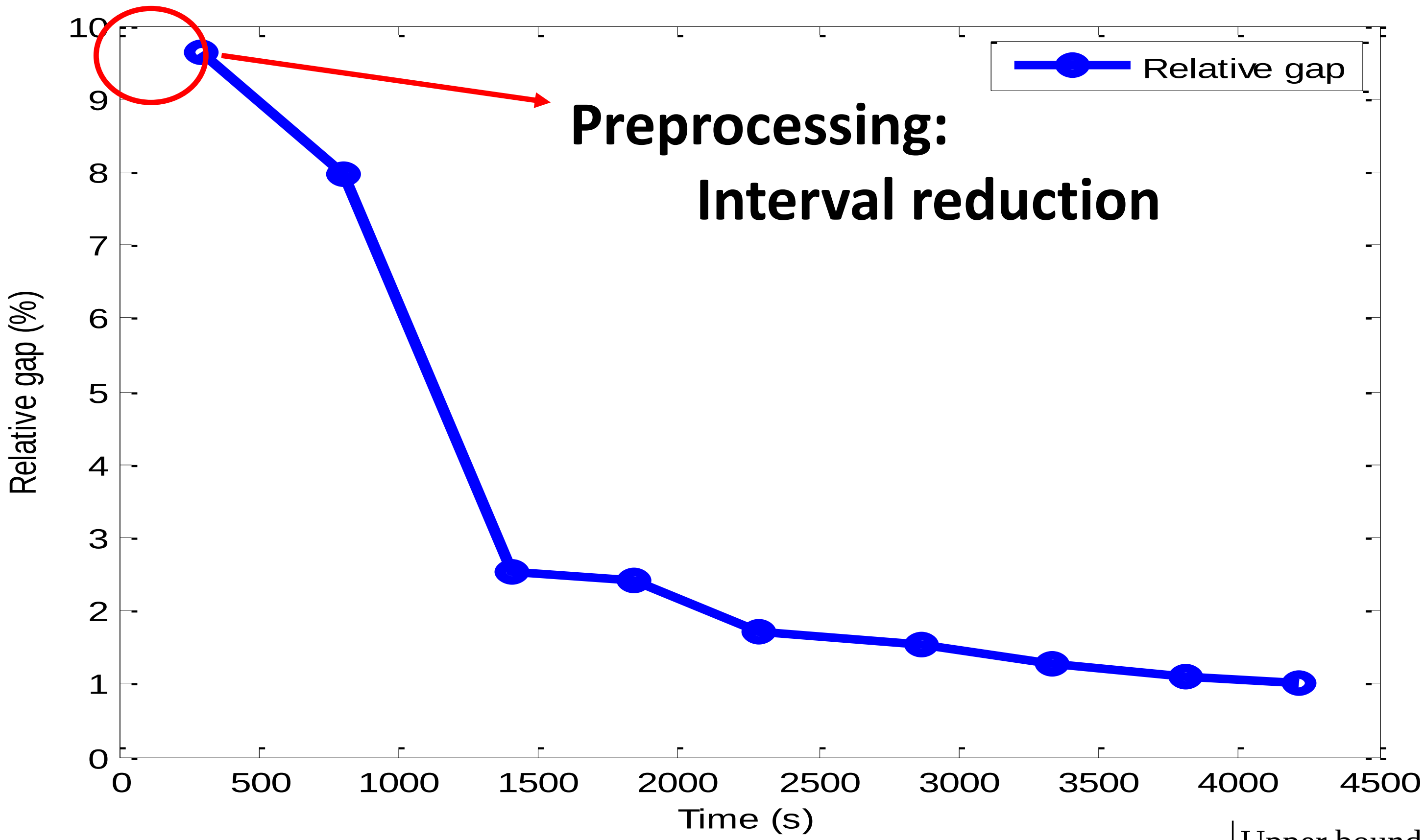
8452 continuous variables

8032 bilinear terms

3456 signomial terms: $K\left(T_j\right)^{7}$ $K\left(T_j\right)^{6.9}$ $K\left(T_j\right)^{4.25}$

Nominal Scenario

- Difficult global optimization problem (ANTIGONE)



$$\varepsilon = \text{Relative gap} = \left| \frac{\text{Upper bound} - \text{Lower bound}}{\text{Upper bound}} \right|$$

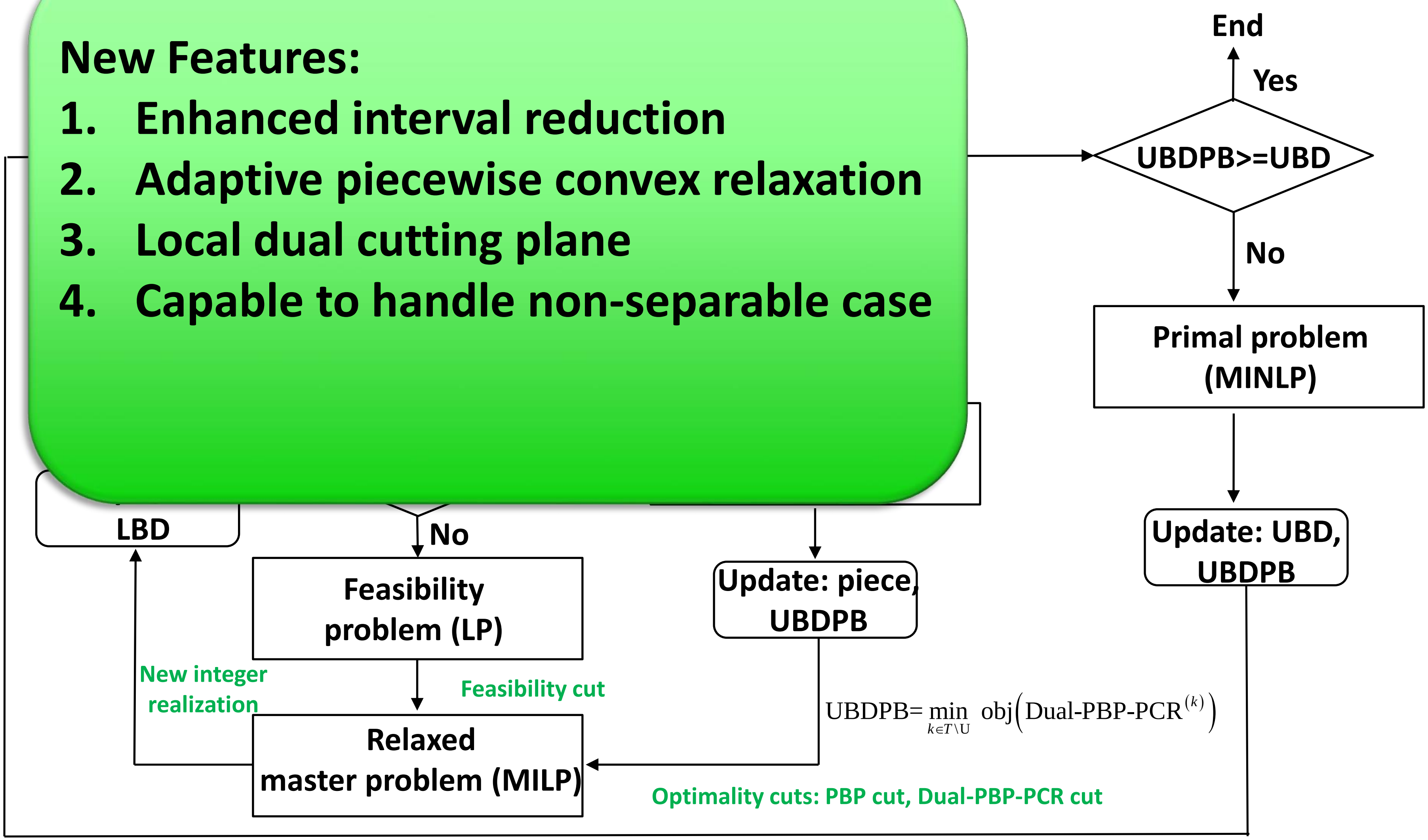
(ANTIGONE: global optimization software, Misner & Floudas)

(Yang & Barton, ADCHEM, 2015)

Optimization: NGBD

New Features:

1. Enhanced interval reduction
2. Adaptive piecewise convex relaxation
3. Local dual cutting plane
4. Capable to handle non-separable case



Comparison

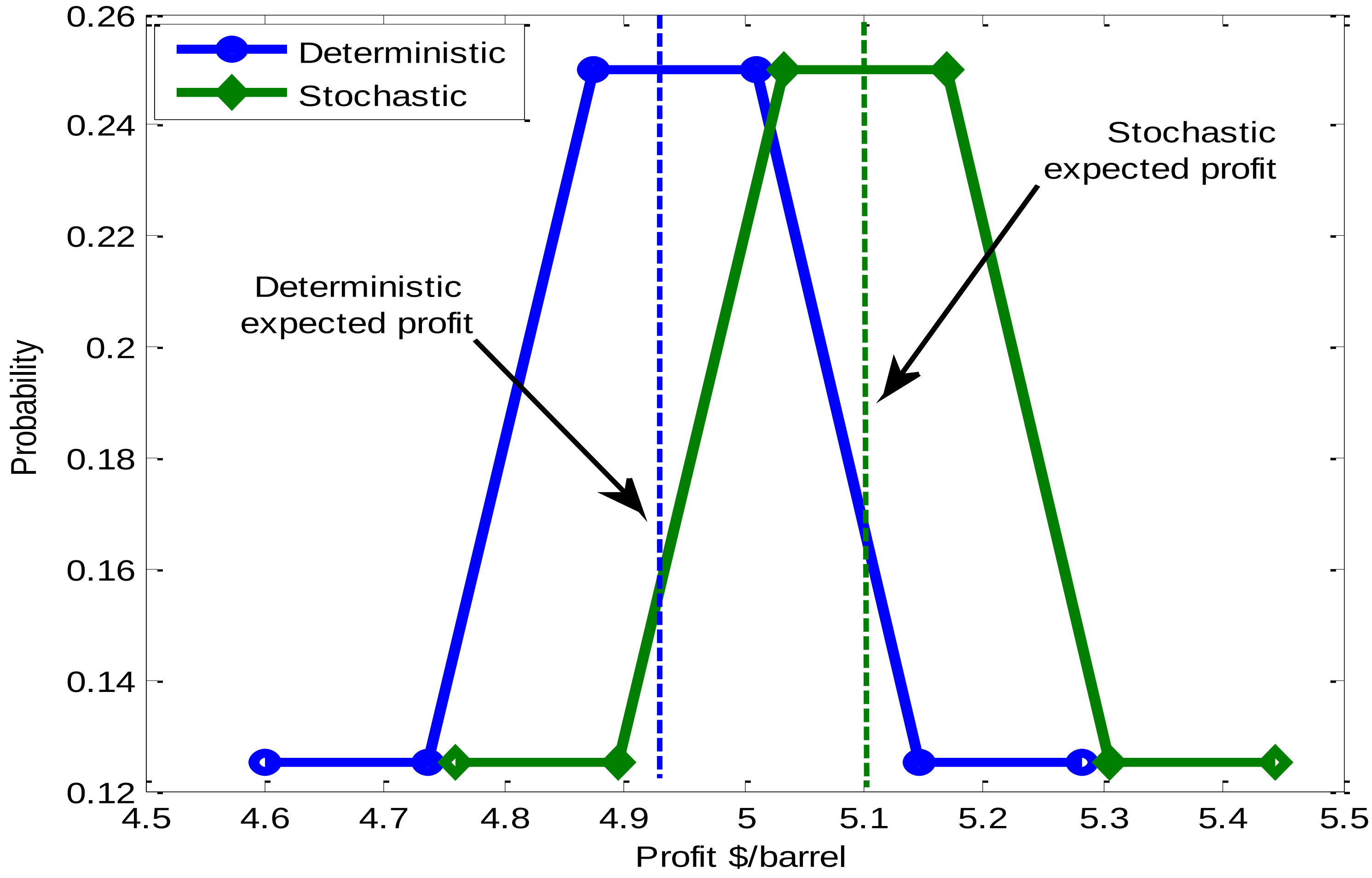
◆ Computational time

Time/Iterations	Adaptive[1]	Non-adaptive[2]
PBP (s)	3932	3896
Dual-PBP-PCR (s)	41572	87009
PP (s)	10686	6094
RMP (s)	148	73
Total time (h)	15.6	26.9
Iteration	126	95

1. Add binary variable if relaxation gap >=10%.
2. Use 6 binary variables for piecewise relaxation .

Results:

- Profit distribution ($/barrel)



Summary

- ***Research Scope:*** Advanced process control/Scheduling for complex processes under uncertainties

- ***Methods:*** MPC, reinforcement learning, spectral method, two-stage stochastic programming (NGBD).

- ***Applications:*** Rod pump, refining process design, CSTR control

Supplement

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PDE Control: Model Reduction

- Parabolic PDE (Example: 2nd order)

$$\frac{\partial x(z,t)}{\partial t} = b\frac{\partial^2 x(z,t)}{\partial z^2} + c\frac{\partial x(z,t)}{\partial z} + dx$$

$$x(0,t) = u(t), \quad \frac{\partial x(1,t)}{\partial z} = 0$$ **Boundary condition**

$$y_c(t) = \int_0^1 x(z,t)\delta(z - z_c)\, dz$$ **Point-wise output**

$$x(z,0) = x_0(t)$$ **Initial condition**

$$|\dot{u}(t)| \le 15$$ **Input constraint**

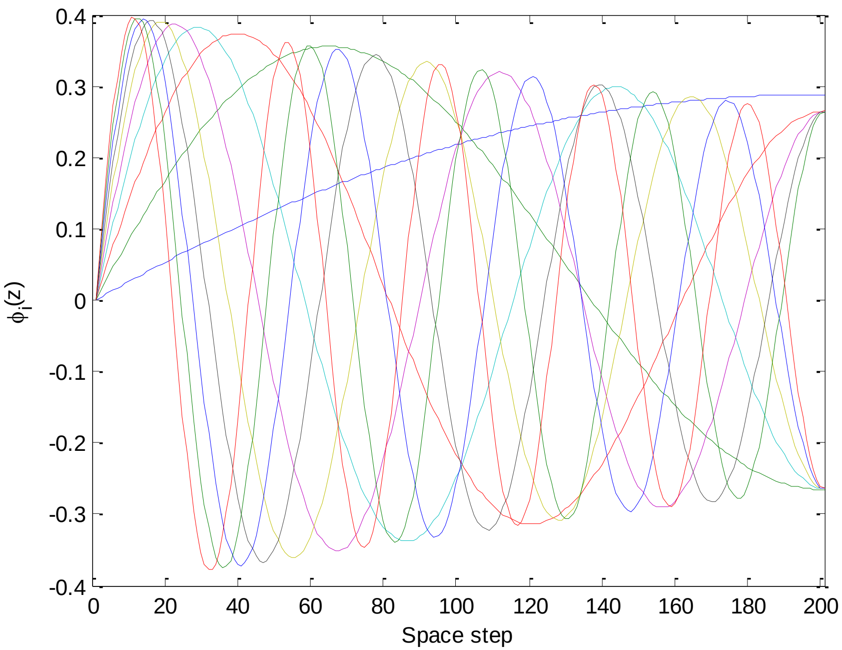
- Eigenfunction analysis

$$\mathfrak{A} := b\frac{\partial^2}{\partial z^2} + c\frac{\partial}{\partial z} + d$$

Solve $\mathfrak{A}z = \lambda_i \phi_i(z)$

Let $f_i(z) = \phi_i(z)$, then

$$x(z,t) \approx \sum_{i=1}^{n_s} a_i(t)\phi_i(z)$$



PDE Control: Model Reduction

- Decomposition & Boundary transformation:

$$a = \left[a_u, a_s, a_f\right],$$

a_u : unstable modal state

a_s : stable slow modal state

a_f : stable fast modal state

$$\dot{a} = \begin{bmatrix} \lambda_1 & & & & & & & \\ & \ddots & & & & & & \\ & & \lambda_{n_u} & & & & & \\ & & & \lambda_{n_u+1} & & & & \\ & & & & \ddots & & & \\ & & & & & \lambda_{n_u+n_s} & & \\ & & & & & & \ddots & \\ & & & & & & & \lambda_\infty \end{bmatrix} \begin{bmatrix} a_u \\ a_s \\ a_f \end{bmatrix} - \begin{bmatrix} \int \phi_1 B dz \\ \vdots \\ \int \phi_{n_u} B dz \\ \int \phi_{n_u+1} B dz \\ \vdots \\ \int \phi_{n_u+n_s} B dz \\ \vdots \\ \int \phi_\infty B dz \end{bmatrix} \dot{u}$$

$\lambda_1, \cdots, \lambda_{n_u} > 0, \quad \lambda_{n_u+1}, \cdots, \lambda_\infty < 0 \quad a_f(t)$ is bounded

PDE Control: MMPC

- Modal model predictive control:

(Assume $x(z,t)$ **can be measured)** → a_u, a_s **are known in real time**

$$\min_{u,\mu} \int_{t=0}^{t_f} y_s^{\mathrm{T}} Q y_s + u^{\mathrm{T}} W_1 u + \mu^{\mathrm{T}} W_2 \mu$$

- **Control performance**

$$\begin{aligned}
\text{s.t.} \quad \dot{a}_s &= \Lambda_s a_s + B_{1s} u + B_{2s} \mu, \\
\dot{a}_u &= \Lambda_u a_u + B_{1u} u + B_{2u} \mu, \\
y_u &= C a_u \phi_u + B_{1u} \dot{u} + B_{2u} \dot{\mu}, \\
y_s &= C a_s \phi_s + B_{1s} \dot{u} + B_{2s} \dot{\mu}, \\
y_{\min} &\le y_s + y_f + y_u \le y_{\max}, \\
|u| &\le \bar{u}, \quad |\mu| \le \bar{\mu}, \quad a_s \cup a_u \in \mathbb{R}^7, \\
|y_f| &\le C \bar{a}_f \bar{\phi}_f + \bar{B}_{1f} \bar{\dot{u}} + \bar{B}_{2f} \bar{\dot{\mu}}
\end{aligned}$$

- **Output for unstable modal states** (← y_u)

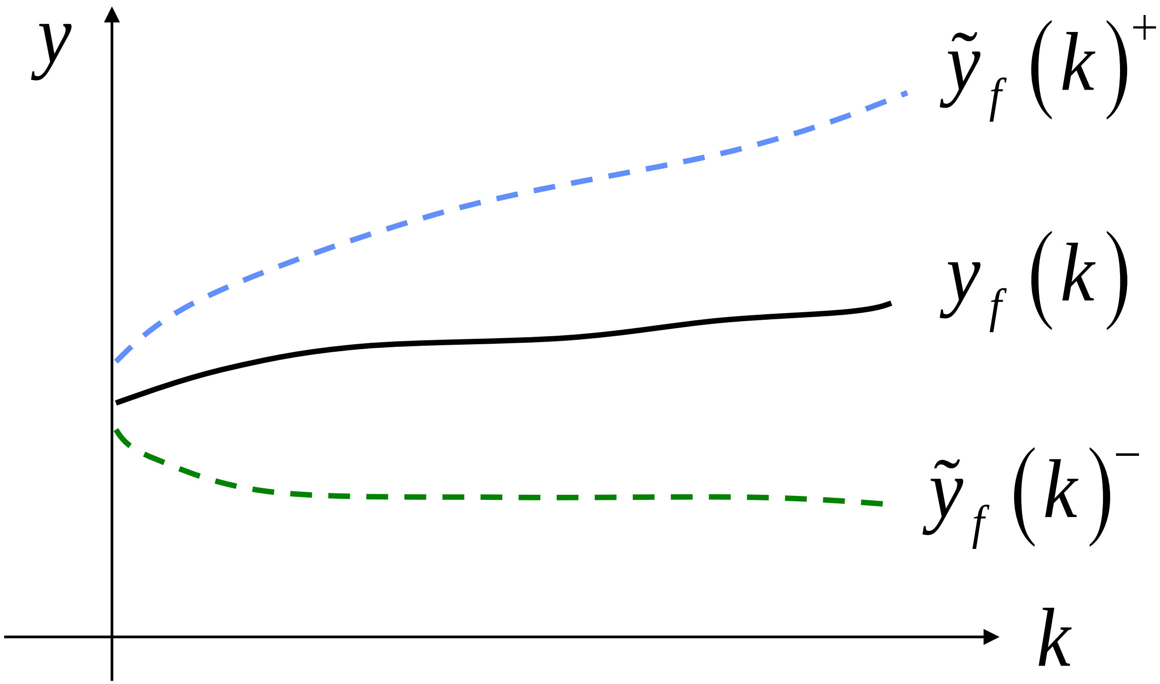
$y = x(z_c, t)$ **output** (← $y_s + y_f + y_u$)

- **Static bound of the output for fast modal states** (← $|y_f|$)

PDE Control: Controller & Observer

- Dynamic upper and lower bounds for: $y_f(k)$

$$\tilde{y}_f(k+1)^- \le y_f(k+1) \le \tilde{y}_f(k+1)^+$$



PDE Control: Controller & Observer

- Solve linear matrix inequality (LMI): K_u, G_u, L_u

$$\begin{bmatrix} S_1 & 0 & [K_u, G_u]^{\mathrm{T}} \\ 0 & S_2 & -K_u^{\mathrm{T}} \\ [K_u, G_u] & -K_u & \dot{u}_{\max}^2 I \end{bmatrix} \succ 0,$$

$$\Xi_1^{\mathrm{T}} S \Xi_1 - S \prec 0, \quad \Xi_2^{\mathrm{T}} S \Xi_2 - S \prec 0.$$

where

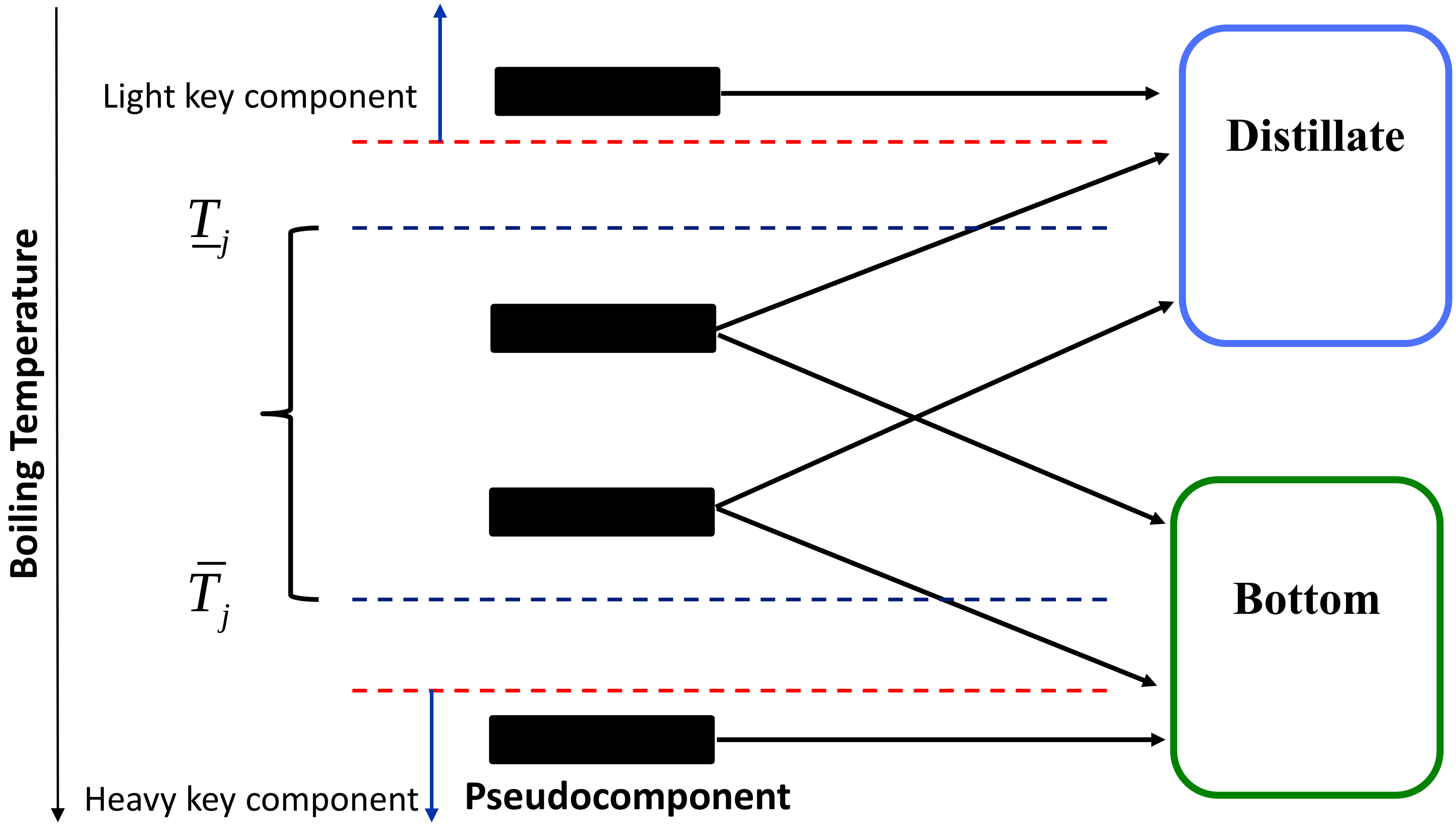
$$\Xi_1 = \begin{bmatrix} \begin{bmatrix} \bar{\Lambda}_u & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \bar{\mathfrak{B}}_u \\ \Delta \end{bmatrix} [K_u, G_u] & -\begin{bmatrix} \bar{\mathfrak{B}}_u \\ \Delta \end{bmatrix} K_u & 0 \\ 0 & \bar{\Lambda}_u - L_u \bar{\mathfrak{C}}_u & -L_u \\ \chi [K_u, G_u] & -\chi K_u & \theta \end{bmatrix}, \qquad \Xi_1 = \begin{bmatrix} \begin{bmatrix} \bar{\Lambda}_u & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \bar{\mathfrak{B}}_u \\ \Delta \end{bmatrix} [K_u, G_u] & -\begin{bmatrix} \bar{\mathfrak{B}}_u \\ \Delta \end{bmatrix} K_u & 0 \\ 0 & \bar{\Lambda}_u - L_u \bar{\mathfrak{C}}_u & -L_u \\ -\chi [K_u, G_u] & \chi K_u & \theta \end{bmatrix},$$

Convergence: $\lim_{k \to \infty} x(z,k) \to 0.$

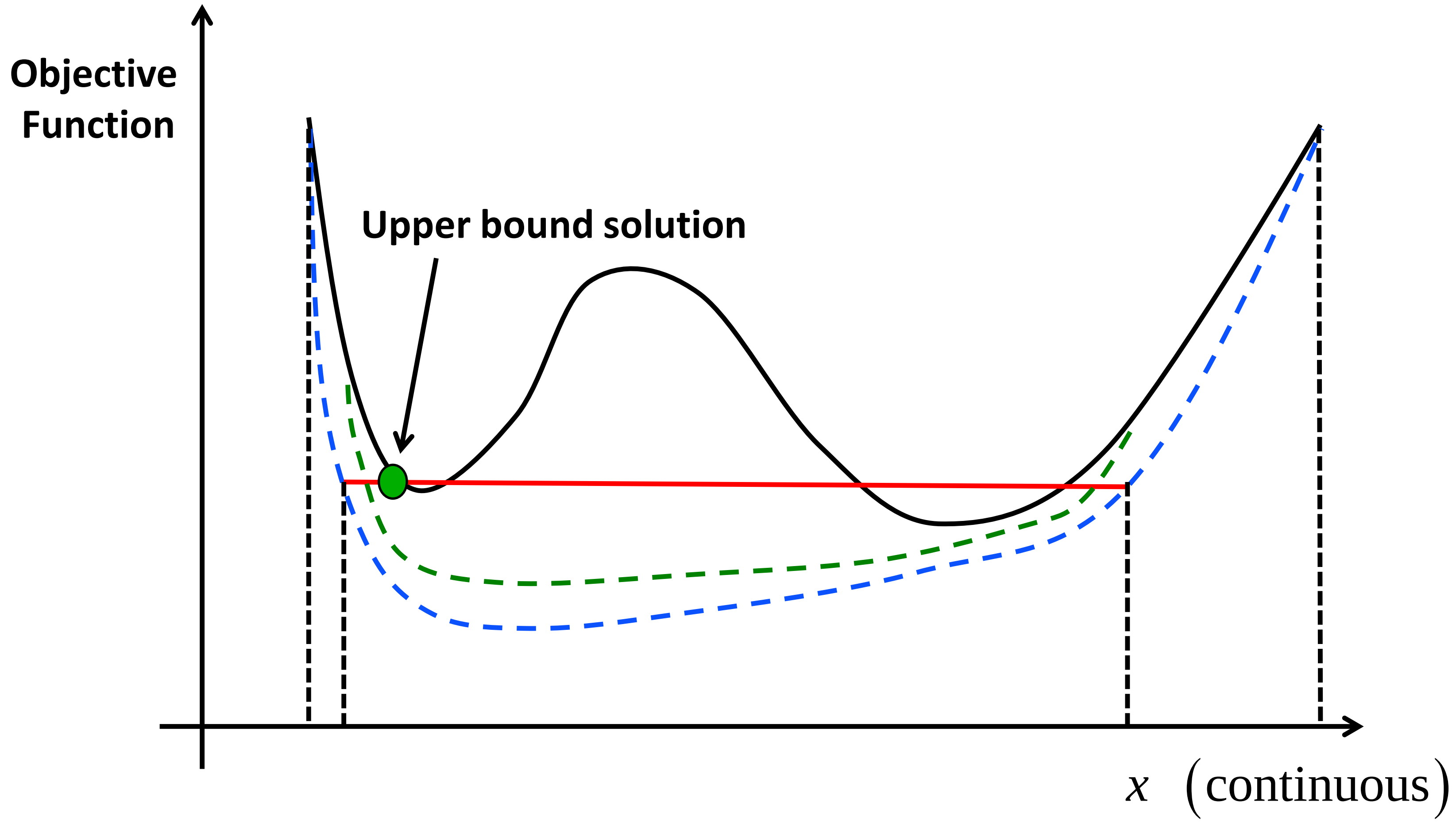
Feasibility: LMI is guaranteed to be feasible by increasing the dimension of a_s

Model 2: CDU Model

- Pseudocomponent method:

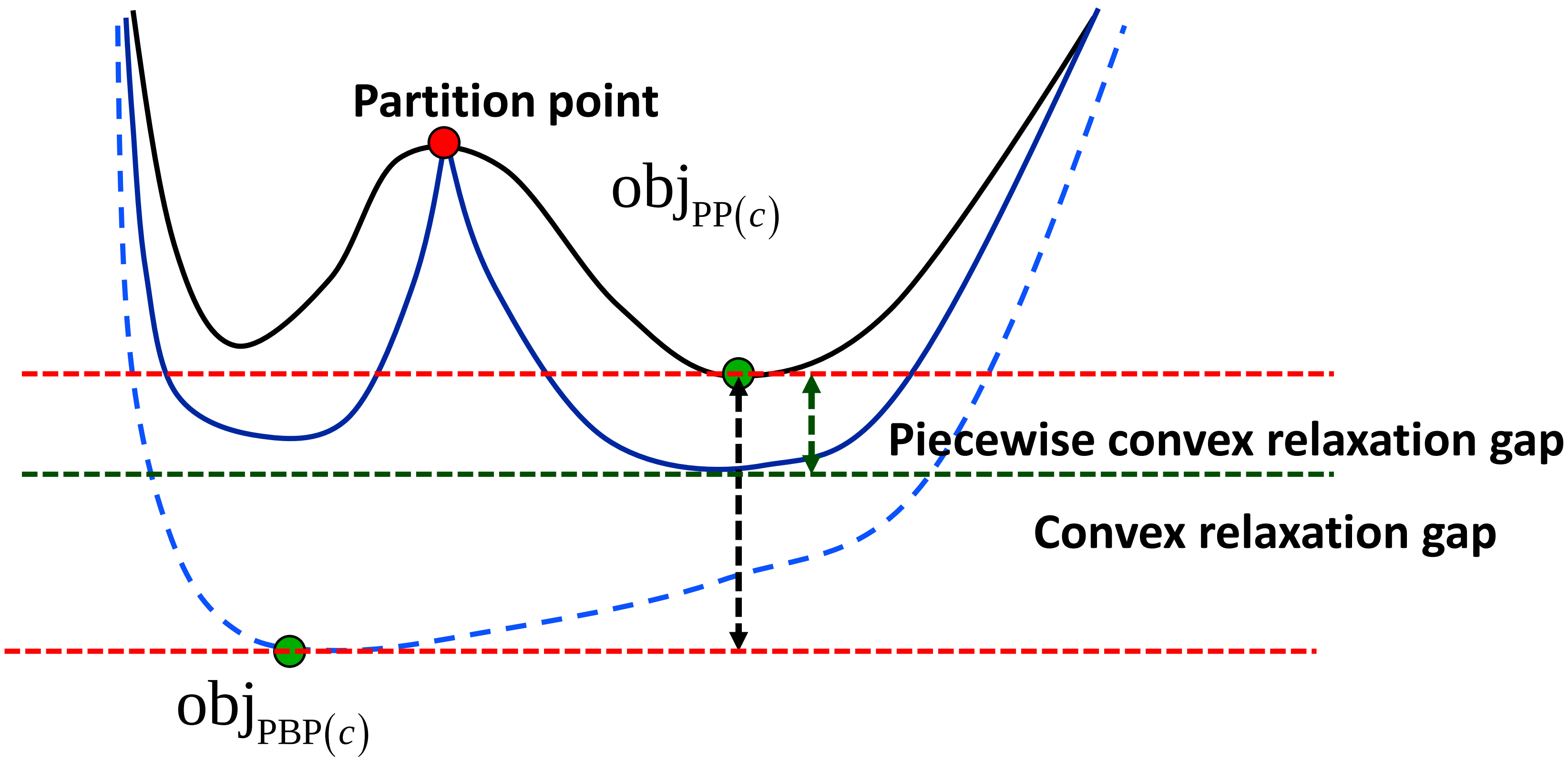


Optimization: Range Reduction



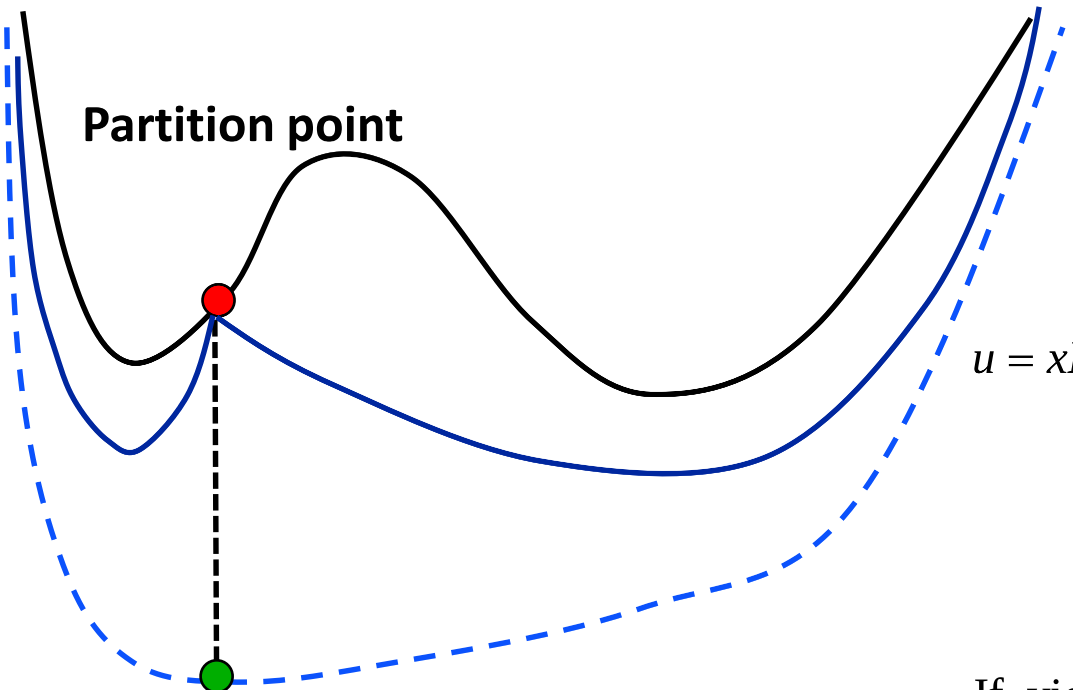
Optimization: Relaxation

- Convex/Piecewise convex relaxation



Optimization: Piecewise Relaxation

- How to select the partition variable/point to get tighter cutting plane?



$$u = xP \xrightarrow{\text{Relaxation}} u \geq x_{lo}P + xP_{lo} - x_{lo}P_{lo}$$

$$u \geq x_{up}P + xP_{up} - x_{up}P_{up}$$

$$u \leq x_{lo}P + xP_{up} - x_{lo}P_{up}$$

$$u \leq x_{up}P + xP_{lo} - x_{up}P_{lo}$$

If violation ratio $= \left|\dfrac{u - xP}{xP}\right| \geq$ threshold,

then one variable in this term is partitioned.

Optimization: Local Cutting Plane

- Objective: design the dual cutting plane only valid in a specific region $\left[\underline{c}_p, \overline{c}_p\right]$.
- Benefits:

Note: $c_p = \left(20 d_p + \sum_{t=1}^{9} 2^{t-1} D_{p,t}\right) Q \rho_p$

1. Better bounds for convex relaxation.

$$f_{p,i} = y_{p,i} c_p,\ c_p \in \left[\underline{c}_p, \overline{c}_p\right] \Rightarrow f_{p,i} \in \left[\underline{y}_{p,i}\underline{c}_p,\ \overline{y}_{p,i}\overline{c}_p\right]$$

2. Extra tight constraints for Dual-PBP-PCR.

$$\sum_{i \in \Omega} f_{p,i} = c_p,\ c_p \in \left[\underline{c}_p, \overline{c}_p\right] \Rightarrow \sum_{i \in \Omega} f_{p,i} \in \left[\underline{c}_p,\ \overline{c}_p\right]$$

Optimization: Local Cutting Plane

- Enhanced PBP-PCR/Dual-PBP-PCR

$$\min_{x,\delta,y,f,b} \quad w(x_s)$$

$$\text{s.t.} \ \sum_{k\in\Omega} f_{p,k,s} = \left(20d_p^* + \sum_{t=1}^{9} 2^{t-1} D_{p,t}^*\right) Q\rho_p,$$

$$\sum_{k\in\Omega} f_{p,k,s} \in \left[\underline{c}_p,\ \overline{c}_p\right],$$

$$G\left(x_s, y_{p,j,s}, f_{p,i,s}, \delta_s\right) \le 0,$$

$$b_{p,i,j,s} \ge \overline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \overline{y}_{p,i,s} - \overline{f}_{p,j,s} \overline{y}_{p,i,s},$$

$$b_{p,i,j,s} \ge \underline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \underline{y}_{p,i,s} - \underline{f}_{p,j,s} \underline{y}_{p,i,s},$$

$$b_{p,i,j,s} \le \overline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \underline{y}_{p,i,s} - \overline{f}_{p,j,s} \underline{y}_{p,i,s},$$

$$b_{p,i,j,s} \le \underline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \overline{y}_{p,i,s} - \underline{f}_{p,j,s} \overline{y}_{p,i,s},$$

$$\sum_{i\in\Omega} b_{p,i,j,s} = f_{p,j,s}, \qquad \sum_{j\in\Omega} b_{p,i,j,s} = f_{p,i,s},$$

$$\left\{x_s, y_{p,i,s}, f_{p,j,s}\right\} \in \Pi_s,\ \ \delta_s \in \{0,1\}^n,$$

$$i \in \Omega,\ j \in \Omega,\ p \in \Theta.$$

$$\min_{x,\delta,y,f,b} \quad w(x_s) + \sum_{\forall p\in\Theta} \lambda_{s,p} \left(\sum_{k\in\Omega} f_{p,k,s} - \left(20d_p^* + \sum_{t=1}^{9} 2^{t-1} D_{p,t}^*\right) Q\rho_p\right)$$

$$\text{s.t.} \ G\left(x_s, y_{p,j,s}, f_{p,i,s}, \delta_s\right) \le 0, \quad \sum_{k\in\Omega} f_{p,k,s} \in \left[\underline{c}_p,\ \overline{c}_p\right],$$

$$b_{p,i,j,s} \ge \overline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \overline{y}_{p,i,s} - \overline{f}_{p,j,s} \overline{y}_{p,i,s},$$

$$b_{p,i,j,s} \ge \underline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \underline{y}_{p,i,s} - \underline{f}_{p,j,s} \underline{y}_{p,i,s},$$

$$b_{p,i,j,s} \le \overline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \underline{y}_{p,i,s} - \overline{f}_{p,j,s} \underline{y}_{p,i,s},$$

$$b_{p,i,j,s} \le \underline{f}_{p,j,s} y_{p,i,s} + f_{p,j,s} \overline{y}_{p,i,s} - \underline{f}_{p,j,s} \overline{y}_{p,i,s},$$

$$\sum_{i\in\Omega} b_{p,i,j,s} = f_{p,j,s}, \qquad \sum_{j\in\Omega} b_{p,i,j,s} = f_{p,i,s},$$

$$\left\{x_s, y_{p,i,s}, f_{p,j,s}\right\} \in \Pi_s,\ \ \delta_s \in \{0,1\}^n,$$

$$i \in \Omega,\ j \in \Omega,\ p \in \Theta.$$

Optimization: Local Cutting Plane

- Old cut (globally valid):

Variables: d, D

$$\eta_s \geq \text{obj}_{\text{Dual-PBP-PCR}}\left(c^*\right) + \lambda_s^{\mathrm{T}} B c^* - 20BQ\sum_{p=1}^{4}\lambda_{p,s}\rho_p d_p - BQ\sum_{p=1}^{4}\lambda_{p,s}\rho_p\sum_{t=1}^{9}2^{t-1}D_{p,t}$$

- New cut (local valid):

$$\eta_s \geq \text{obj}_{\text{E-Dual-PBP-PCR}}\left(c^*\right) + \lambda_s^{\mathrm{T}} B c^* - 20BQ\sum_{p=1}^{4}\lambda_{p,s}\rho_p d_p - BQ\sum_{p=1}^{4}\lambda_{p,s}\rho_p\sum_{t=1}^{9}2^{t-1}D_{p,t}$$

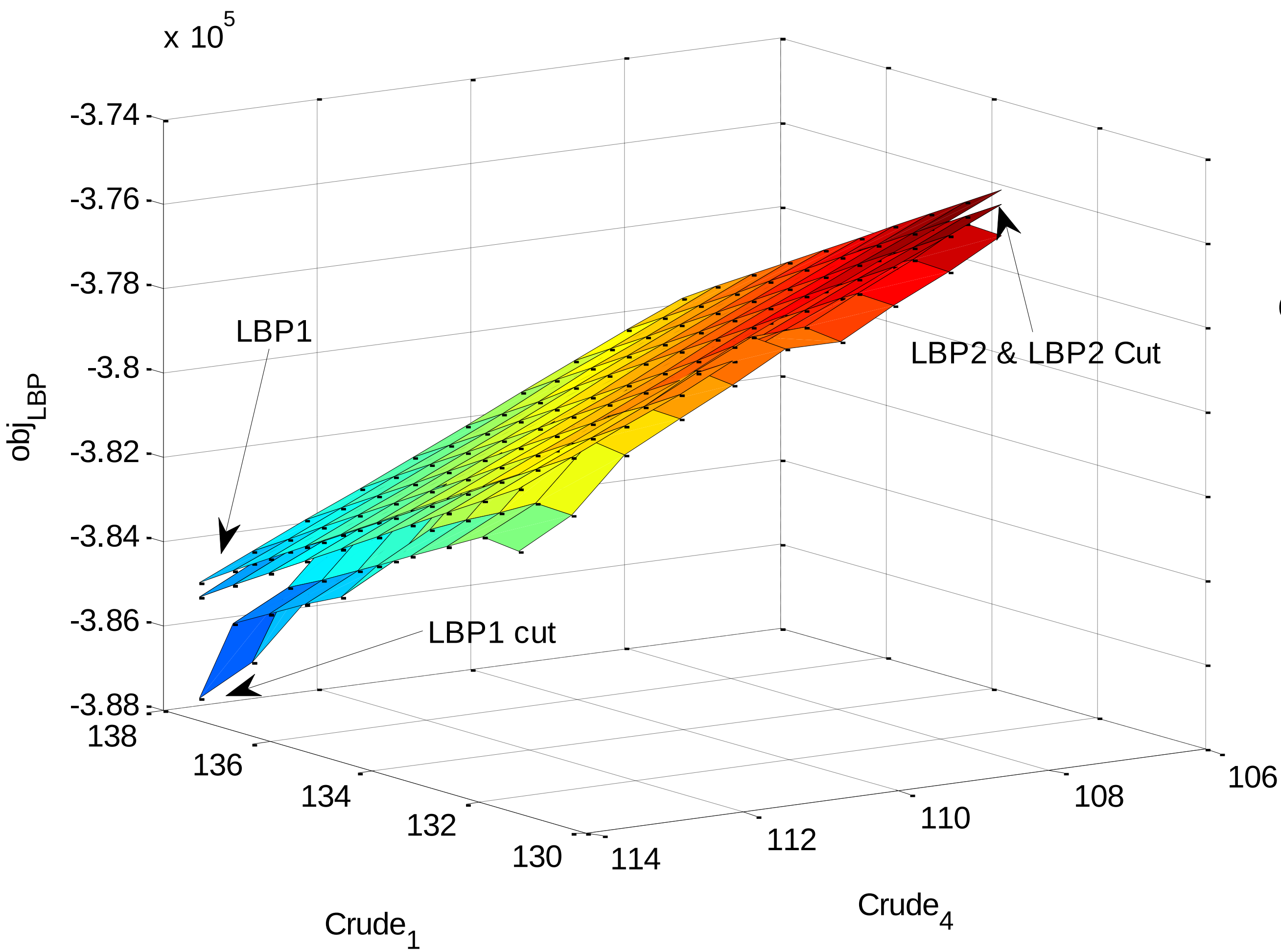
$$+\sum_{p=1}^{4}M_s\sum_{h\in\Upsilon, D_{p,h}^*=0}D_{p,h} + \sum_{i=1}^{4}M_s\sum_{h\in\Upsilon, D_{p,h}^*=1}\left(1-D_{p,h}\right)$$

$M_s < 0,$ can be determined systematically

Note: $\text{obj}_{\text{E-Dual-PBP-PCR}}\left(c^*\right) \geq \text{obj}_{\text{Dual-PBP-PCR}}\left(c^*\right)$

Optimization: Non-separable case

- Bilinear term=stage I variable × stage II variable



LBP1: convex relaxation
LBP2: Reformation + convex relaxation